Evolving Systems
Recognition and Description

Mario Ludovico[1]

Abstract

This paper tackles the issue of describing and analyzing complex evolving systems for practical purposes. At variance with other definitions of "complex system", which usually focus on *agents* that show a spontaneous capacity of self organization and adaptation, the systems considered in this paper are those which the observer describes focusing *only* on the *interactions* occurring between any set of agents. The approach to the subject is of a probabilistic nature, while the use of mathematics aims also at stressing the determinant influence of human languages in observing events as well as the context of their occurrence.

The contents of this paper re-propose – in a more concise form – a theory I have expounded first in 1988 in a volume titled "*L'evoluzione sintropica dei sistemi urbani*" [2], published in Italian language only. Still today, as it was at that time, a related issue is the practical difficulty of displaying the theory properly in the size and format of a paper to be sent for publication to specialized magazines. The contents of the theory cannot conveniently be compressed in a *paper*, however reasonably long the paper may be. Despite this conviction of mine, I have tried (albeit without noticeable success) to communicate *also* through the publication of relevant papers in English, both in the Internet,[3] and in one international conference[4], trying to make the theory escape the communication boundaries imposed by the use of Italian language. I am trying hereby again.

During my professional activity spent across the world, I have had a few opportunities to test the viable application of the theory to the solution of real problems. Thus, while aging, the theory has

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In 1988 the Italian government awarded him the Prize of Culture for one of his books, in which he expounded the theory that is more concisely re-proposed in English in this paper. On various occasions, sections of the theory have been applied for coping with regional planning problems. E-mail; mario.ludovico@tin.it
[2] Bulzoni Publisher, Roma 1988 (1st edition) and 1991 (2nd edition)
[3] www.mario-ludovico.com/pdf/syntropy.pdf, (2005); **Syntropy: Definition and Use**, online magazine "Syntropy Journal" (www.syntropy.org/journal, 2008).
[4] **Towards a Complex Systems Economics**, Simon Kuznets International Symposium in Kiev, Ukraine, 25-28 May 2011, Proceedings.

been corroborated by experience. Perhaps, this is an additional reason why I deem it worth trying again a better diffusion of the theory through the publication of this rather long paper in English.

Nevertheless, the theory remains only one of the many rationalizing attempts whose fate is – because of the intrinsic nature of the subject addressed – to gravitate still out of the perimeter of science. In this connection, another reason for recalling attention to this way of modelling complex evolving systems is the (probably vain) attempt to oppose the overwhelming fashion of computational simulations, which are increasingly encroaching on the hard beaten field of more consistent researches that strive to understand complex systems. Skilled and well equipped computer programmers can help simulate anything and "prove" any arbitrary theory.

In my opinion, whatever the destiny of works like this one, any self-consistent theory must be based on a coordinated set of clear and appropriate fundamental concepts, accurately defined and – as far as possible – unambiguously expressed and developed. This should be the main purpose of availing oneself also of mathematical language.

This paper is not an easy reading, and it is certainly boring, as it is of most theoretical works. The interested reader should persevere at least through the first three chapters, before obtaining - in the fourth chapter - a significant picture of the theory and of its possible use. What I can do in return for any generous intention to undertake such an effort is only to assure the reader that this paper is not one of the many commentaries on theories and ideas expressed by other authors about subjects of the kind, but the re-presentation of a substantially original theory.

Contents

Introduction

Who decides to read a paper like this is either somehow familiar with the subject at which the title of this paper hints or is an educated beginner that has been charmed by terms like *complex systems, emergence, self-organizing and adaptive systems, evolution* and other akin terms. Such terms have since decades projected new hopes and ambitions on the screen of our imagination, though – regrettably – have favoured the rise of novel illusions rather than the development of the human knowledge about effective means of control on complex phenomena of the real world.

The definition of "complex and adaptive system" is the prime subject of discussion, which this paper faces in its first chapter. A commonly shared idea is that complex systems are characterized by a large number of *interacting agents* whose actions, in consequence of a self-organizing collective behaviour, lead to *emerging states* of the system, which are difficult or impossible to predict on the basis of the agents' individual behaviour. The concept of "agent" is quite general, as it may regard either the individual active components or particular sets of active components that form a system. Moreover, in complex adaptive systems most of the individual behaviours are not subjected to a central control.

The long lasting fashionable rush of many scholars and students towards charming horizons, above which shines also a constellation of theories on complex systems, has found a fertile ground and a powerful thrust in the wonderful and explosive development of both computer technology and computational languages.

Once again, it has been possible to philosophers – professionally entangled in the sphere of conceptual complexity – to claim their way to access and manage "positive science" at a level appropriate to their own conceptual world, after the severe frustration undergone by ambitious "non-natural" (or "pseudo-natural") philosophies developed between the XVIII and XX centuries.

Also in a text on complex and adaptive systems, an "academically correct" habit would require a scholarly review of the relevant literature produced during recent decades. Regrettably, this paper is not *academy based,* and does not claim to exhibit "scientific" stuff in search for academic acceptance. To the contrary, the theory expounded in this essay is substantially based on many years of professional experience gained on the field and in various continents, after vain initial attempts to avail myself of valuable academic theories in outlining solutions to serious complex problems met in the real world.[5] The theory of this essay aims only at suggesting a practical method for coping with decision problems inherent in planning or management or governmental activities.

Therefore, I limit myself to a few quick remarks on fashionable theories and models that concern the subject.

The recent amazing development of computational languages and the powerful control on the performances of computers can nowadays – and even more in the near future – allow anyone to represent anything in the desired way. We live in the era of *computational simulations*.

Think, for instance, of the possibility of creating a complete movie fiction story by use of computer only. Together with all the necessary and useful input data for the computer work, one might even use, as an additional input to be processed, the photographic images of one's family members as well as the images of one's neighbours plus those of the local shop/market keepers and local municipal agents, together with the images of the relevant residential area and of crowds of "anonymous people", to produce "documentary" movies telling different possible stories, according

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[5] **A** valuable proto-example of comprehensive theory on systems is in the book ***General System Theory. Foundation, Development, Applications,*** **(**George Braziller Inc., New York, 1967), by Ludwig von Bertalanffy (Austrian biologist, 1901-1972). The book is often unfairly neglected. However, also in that theory it seems difficult to find practical methods for the study of *real complex* systems. Bertalanffy's renown bio-growth equation seems seriously questionable too.

to different particular *initial* conditions as well as *future hypothesized* conditions. There is little to doubt: any person could *show* – for example – that an increasing amount of foreigners immigrated from abroad to the area where the person lives (or, alternatively, an increasing diffusion of unemployment) causes a massive displacement of families settled there, **if** these are (amongst innumerable other ones) the theses the person would like to *prove*. It seems difficult denying the persuasive effect on the addressed audience.

Such actual possibilities should be viewed as a clear indication of the misleading potentials of computational simulations and – therefore – as a warning to all those who incline to trust the reliability of the *computational* approach to the study of any kind of "complex system". As an academic "scientist", however, one would refrain from using movie images, not only to avoid unnecessary high costs, but also in the awareness that – from an academic standpoint – the results of his computational modelling would appear more science-like and credible if presented through orderly sets/sequences of numeric figures, mathematical formulas and appropriate diagrams.

The computational approach through movie images is instead more frequently preferred in cosmological and climate simulations, in the attempt to make rather arbitrary theories more credible.

What is clear is that wherever scientific experimentation and relevant replicable experimental check is impossible – *i.e.,* in absence of a scientific support - computational simulations can provide theories of any kind with a consolatory performance together with brilliant exposition and propaganda means.

"Often, tools get mistaken for theories with unfortunate consequences; elaborate computer programs (perhaps with lovely graphics) or mathematical derivations are occasionally assumed to take a scientific statement, regardless of their scientific underpinnings. Indeed, entire literatures have undergone successive refinements and scientific degradation, during each generation of which the original theoretical notions driving the investigation are crowded out by an increasing focus on tool adeptness. This often results in science that is smart but not wise". [6]

As a simple conclusion, I wish soon to remark that the unpredicted advances in computational languages and computer performances have also brought about an effect which is the opposite of the expected one: Instead of making us aware of the *determinant* (and often misleading) constraints imposed by human languages on the human effort to understand the real world, many of us do now take the performances of our languages as the *real* behaviour of the world around and inside us. It seems to me that it is an additional triumph of metaphysics (and self-deception) over scientific knowledge.

Of a minor impact on the large public, but still a reason for pride with many scholars devoted to "complex systems", is an ample set of specific mathematical methods that are used for various particular purposes, especially in the variegated field of the so called "operational research". Such methods have undergone an important development with the development of mathematics applied to the self-correcting or self-controlling mechanisms that are activated through complicated *feedback systems*. The showiest results of this particular mathematical and industrial technology is in the field of robotics, while a number of other systematic theories, such as those concerning graphs, thermodynamics, dynamical systems, stochastic processes, games, fractals, catastrophes, chaos, networks of various kinds, etc., can usefully be applied to some complex real world aspects, with a view to carrying out schematic *objective* analyses and making *calculable* predictions.

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[6] John H. Miller, Scott E. Page, **Complex Adaptive Systems. An Introduction to Computational Models of Social Life**, Princeton University Press, 2007, pp. 4-5. The book's pages are often ornamented with smart and wise sentences like that one. Pity, however, I could find there neither smart nor persuasive application examples of computational models of complex adaptive systems.

Most of the models drawn from the above mentioned mathematical theories regard the way in which different complex systems may achieve equilibrium states or transform their structure into a different one.

A major limit of this kind of approach in addressing biological and socio-economic systems consists in forcing the features of the observed real world into the pre-made conceptual schemes proper to the theories applied, in order to make the relevant equations and algorithms usable. In most cases, pre-made mathematical models can be applied to very particular and limited aspects of the study subject, so as to avoid the implication of higher degree of complexity inherent in the same subject.

Intrinsic additional limits of some mathematical models are in their deterministic vision of the study issue, whereas the data used for representing large socio-economic and biologic systems are generally of a stochastic nature.

Usually, deterministic models of complex systems require the adoption of a number of "constant" parameters, which are in most cases of difficult determination and often sources of incorrigible ambiguity as to the significance of the models' performance. A slight change in the value of some – or one only – of such *constants* may surprisingly twist the performance of the model from one application to another, *coeteris paribus*.

Just as one example, amongst an ample variety of possible examples, consider the theory of the random Boolean networks. (Despite the word "random" the theory is of a deterministic kind). It's a theory that – to some extent - can usefully represent dynamical systems whose *agents* are identifiable in two possible *states* only, dubbed - for instance - "1" and "0", or "positive" and "negative", "active" and "idle", or "black" and "white", etc.; each variable (agent) may change its state in a dynamic process.

In the theory, such agents are said "binary variables". If these variables form a set of interrelated *binary* components, the set can be represented as a Boolean network. The network results from the fact that each binary variable (or "binary agent") is *controlled* by one or more other binary variables and, in its turn, *controls* one or more other binary variables.

Given any large set of binary variables along with the network of the relevant *control* connections (each variable being initially caught in *one* of its *two* possible states), the Boolean system may be viewed as in its *initial configuration*. The *constant parameters* of the modelled system are - in this case – the numbers of the *control connections* (termed "connectivity" by the theory) relevant to the binary variables (or binary agents). The theory provides the rules apt to simulate the transition from any given initial configuration of a random Boolean network to a subsequent one, each transition step thought of as occurring in a discontinuous time unit.

Considering that neural networks consist of neural cells that are inter-connected through synapses, which may alternatively be "active" or "idle", it is easy to imagine, for instance, the possible applications of Boolean network models to the study of neural processes. Actually, Boolean networks have been used also for modelling gene expression and protein regulation networks.[7]

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[7]   Biologist Stuart A. Kauffman (***The Origin of Order. Self-Organization and Selection in Evolution***, Oxford University Press, 1993) and followers have used random Boolean networks in the study of possible evolutions of gene molecular chains. Apart from licit remarks on artful constraints imposed on the "Boolean networks" of the simulations, one major limit consists in considering the modelled system as primarily isolated from "the rest of the universe". (The same limit regards most mathematical models of socio-biological systems). Early appropriate comments on that book, amongst the several ones published later, are by U. Bartocci, ***Le origini dell'ordine***, in www.cartesio-episteme.net, 1994.

As to *non-deterministic* models of complex dynamic systems (the so-called *stochastic processes*)*,* of a major interest are probabilistic models apt to describe possible *evolution processes* of systems characterized by *probability states* of the respective components, be the components defined as *agents* or not. Models based on *Markov chains*, for example, are powerful analytical instruments used in a wide range of simulations concerning dynamical systems whose salient features are caught in the *probability states* inherent in their sequence of interconnected configurations. Practical problems in adopting models based on Markov stochastic processes may arise in choosing the appropriate probability distributions, as it is necessary for starting reliable simulations.

Of a remarkable interest are also the *Poisson stochastic processes*, which find useful application to particular chains of "random" events that depend on time.

The theory expounded in this paper is based on a probabilistic approach.

The basic philosophical assumption on which the theory rests is that the recognition and the description of any "system" are *subjective* mental operations. The theory describes *a representation* of alternate *unstable equilibrium states* and *sequences of transition phases* in system evolution processes, which entails series of *transformations* of the dynamical structure of complex evolving systems. The theory includes conceptual instruments for significant *quality* evaluation of each particular state characterizing the *represented* evolution of a complex system.

The theory was conceived to express in a simple mathematical form one particular *mental representation* of the processes that are observed-in or thought-of socio-economic systems. Initially, the theory aimed at describing and analysing *the representation* of both the activities and the development processes proper to observed interconnected human settlements or to systems of interrelated economic sectors. However, a number of "specialized" models can also be derived from the theory for the study of other kinds of *conceptualised* complex systems.

The theory is the expression of a mental representation that focuses attention on the intensity of the *interactions* characterizing particular aspects of the observed system, rather than focusing on the behaviour of the "agents" that activate the system. One of the theory's basic philosophical assumptions is that the *interactions detected* are the vehicles of almost all of the *specific* information needed by the observer to better understand the processes studied. It is obviously a conceptual simplification, which can nevertheless corroborate the effectiveness of the representation.

The identification of the conceptualised system depends on the *specific* nature of the interactions that the observer can single out from a larger context of observed events. Such interactions must be measurable by use of significant measurement units, in view of analyses and calculations that are useful for practical purposes. The *intensity* of each detected interaction may be expressed in *relative terms* with respect to the overall amount of the interactions detected in the system during a conventional time unit**;** which actually results in a set of *percent interactions* recorded by the specific surveys carried out on the studied system.

The *active components* (the "agents") of the system are recognized and characterized according to the *nature* of the respective interactions.

The system's set of active components *does always include* one *non-characterized* "external component", which represents the *rest of the universe* with respect to the particular set of the other components described. Thus, for instance, if the represented system regards vehicular traffic between the traffic origin and destination localities of a given set, then the "external component" of the system is both the destination *and* the origin of the traffic flows bound-for or coming-from any *unidentified* localities that do not belong to the given set of well identified localities. Note that, in this example, the traffic flows are the *interactions*, while the flow origin *localities* themselves are considered by the theory as the *activators* (the "agents") of the system.

Since – in all cases - the system of the detected interactions depends on the survey's method and time, the recorded *percent interactions* are viewed by the analyst as the *interaction probabilities* concerning a *conventional original* configuration of the studied system, which simply and necessarily corresponds to the state of the interaction distribution at the survey time.

1. The system to recognize and describe

The mental activity of defining systems is in itself a complex system of operations. In dealing with "systems", humans are not always aware of the limits and implications inherent in the way they recognize and describe the systems considered.

1.1 *Real world and its representation*

Let us start with a banal example: any active technological contrivance constructed by humans for whatever purpose should never be considered as a complex system of interacting components, *unless* one accounts also for its context of necessary connections with the relevant human operators as well as, through them, with the rest of the universe. To mean that the operators and the rest of the universe are *necessary* components of the system, in which the active machine under consideration is only a minor component. In other terms, the contrivance in itself is *not* a "complex system", but only a more or less *complicated mechanism*.

However complicated and sophisticated may any mechanism be, no contrivance in itself can be considered as a complex adaptive system to the extent that it can neither *spontaneously* adapt itself against unexpected adversities nor evolve and transform into something different from which it is originally. Therefore, the *complex systems* that deserve our attention in this paper are those evolving systems whose possible *artificial* behaviour, if any, constitute only quite a minor aspect of the systems' activity.

The manner in which we recognize and identify any "complex system" consists in the description of how, when and where our attention focuses on a very partial set of *evolving* events amongst the innumerable describable events in the universe, in the awareness that the identified system may be thought of as anything but an *isolated and self-sufficient object.*

In this connection, we should obviously suspend any comment on what the *whole universe in itself is*: according to our common sense, we should doubt that the universe is a *contrivance*. The universe, thought of as an unlimited set of events, appears through its observable components as an extremely complex system, which seems somehow evolving from unknown origins towards unknown ends, despite a number of *mechanisms* that contemporary science hypothesises about some of the events observed and – to an almost negligible extent - controlled by humans.

In an apparent contrast with any complicated artificially active mechanism, a significant though "minimal image" of evolving system may be caught in the mental image of a stone. Let alone amazing images of complex systems like that - for example - of a new-born baby that becomes adult, we may in fact think of any stone as of a particular evolving system, if it is thought of as a *thing* that transforms under the effects of climate and other natural actions only. Jumping beyond any temptation to confine the concept of evolving system to the description of social or biological organisms, we should seriously consider that the "stone system", at variance with any artificially working mechanism, can be viewed as a complex adaptive system. The stone's internal relationships between different and – at least in principle – well identifiable atoms and molecules (these being viewed as generators of interactions), undergo alterations primarily because of the

action of the *external component*, which in this case consists of all the variable and unpredictable climate and environmental factors. (Any "image" like that is announcing the importance we may associate with the concept of intrinsic "organization" and/or "disorder" referred to complex adaptive systems). The stone system is "adaptive" in that it *transforms* itself according to necessity, that is, according to the variable causes that constrain or modify the activities of the stone's components. The transformation may involve both the physical and the chemical structure of the stone, in the response to the variable effectiveness of accidental external factors and to the alterations in the stone's internal energy concentration and distribution. In this kind of "evolution", the stone remains *itself*; *i.e.,* it remains an identifiable system despite the changes undergone. Not to add that the *stone* might be a "trunk" of a petrified forest.

Any *artificial* mechanism, instead, ceases its *designed* activity and becomes no more identifiable *as such* if it undergoes *accidents* which are incompatible with its functionality. In no case it can adjust itself to "traumatic" events and transform spontaneously into a different *active* mechanism capable of performing any *significant* function. Any functional mechanism is recognizable as such until it remains subjected to a *central control* that is external to the mechanism itself.

To proceed on with the analysis of the relationship between mental processes and human perception of the real world, we should first of all account for the *language-conceptual* framework within which *any system* is recognized and described.

Our languages are selective, and allow us to recognize only those events which can be expressed in words or by means of other artificial symbolic systems. The exercise of knowledge is the way in which our languages decompose our overall sensorial perception into much smaller *perception units*, in order to facilitate our physical orientation-in and control on a very complex reality.

By use of languages, our mental activity establishes *elementary concepts through which we represent* the universe. "Concepts" are the indispensable primitive *models* that allow us to focus on "aspects" of the real world, which we try to control and within which we strive to survive.

Our knowledge is substantially a *linguistic representation* of the real world (which obviously includes our languages too) and should never be taken as an expression of the world in itself, albeit in most cases our daily experience corroborates the conviction that our linguistic models *coincide* with the *things* we are dealing with. Such a conviction is the source of unceasing delusion and frustration, for our awareness about this point is neither spontaneous nor immediate.

Our *mother tongue* teaches us the use of the *primitive* instrument that is indispensable to start controlling our relationship with our native environment. Our mother tongue is an impersonal *institution* that pre-exists us and our ancestors, which "ever since" dominates the inter-personal connections that bind the individuals of our community together. So we learn to believe that each significant word (noun) of our language coincides with the respective material or immaterial object referred to, for it seems impossible that real *things* are not in themselves the pre-given *institutions* meant by the words and by the other symbols we *naturally* apprehend for designating and representing them.

In other terms, it seems impossible to us that *distinct objects* do not exists *per se* in the same way as the respective *distinct names* exist, and which we use - initially - *to perceive and memorize them* as *different objects* and - later - to recognize and identify them as if they were *clearly* distinct things.

1.2 *Language, representation and science*

This hurried incursion into the sphere of language seems important to me, to remind ourselves that all human mental activities are structured by the languages we have learnt to use.

History is the effect of how different individuals and communities, in different times and places, using different languages and relevant practices, have acted - on the basis of their respective conceptualized experience - in understanding and governing the world of their interest.

We should also remind ourselves that until the first decades of the XX century major philosophers of our Western culture were still entangled in linguistic scrapes, which originated from the study of the connections between languages, meaning, thought, logic and the "real world". One paradigmatic example of the conceptual mess about the formation of human knowledge is, in my view, provided by the *Tractatus Logico-philosophicus* written by Ludwig Wittgenstein in years 1918-1921.[8]

Amongst a few clear manifestations of a new awareness concerning the relationship between language, real world and knowledge, the works published by US anthropologist Benjamin Lee Whorf are - in my opinion - particularly important.[9] After many years devoted to the study of various languages and cultures of indigenous populations of the Americas, Whorf found sufficient arguments for expressing the thesis that human beings do not share an identical innate mental structure, which – according to an erroneous common belief - is independent of the linguistic communication means proper to the communities to which they belong. Instead, Whorf maintains that each linguistic system is a mental paradigm which *per se* determines *what one perceives* in the environment one lives in and what one thinks of it.

Because of the large number of such different mental paradigms, the ways of perception and thought in different human communities, which use different linguistic systems, involve perceptions and visions of the real world that are basically different from one another. This fact implies that not all of the observers who face the same physical evidence will draw a common (or a strictly similar) image from the experience of it, unless they share a common cultural background.

Through an ample range of different languages, human beings have experienced, at different scales and by special uses, various degrees of *effectiveness* of the world's representations they were and are accustomed to believe in.

After centuries of attempts to connect the conceptual representation of the real world with practically *effective* knowledge, the *emergence* of modern science (starting in Europe from the 16th century) has once and for all made it clear that the most effective language consists in tying – *if and where this is possible* - the *meaning* of words/concepts (or other symbols) not to objects or events or – worse – to other concepts, but to precise *operations* on the things one can handle and manage. That is the why and how modern positive experimental science could emerge and develop together with the specific languages adopted by modern sciences.[10]

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[8] The English version of the book was published with this title in 1922.

[9] B. L. Whorf, **The Relation of Habitual Thought and Behaviour to Language**. Written in 1939, it was originally published in "Language, Culture and Personality**:** Essays in Memory of Edward Sapir" edited by Leslie Spier, 1941.
- **Language, Thought and Reality**: **Selected Writings of B. L. Whorf**, edited by John Carroll, John Wiley & Sons, New York, 1956.

[10] In the 20th century, a world-wide epistemological debate on "operationalism" was opened, which focused on the structuring importance of languages in modern scientific knowledge (the latter viewed as a system of achievements *effectively* different from those pertaining to free philosophical thought), after physicist Percy Bridgman published his book **The Logic of Modern Physics** (McMillan, New York 1927). The same physicist (1946 Nobel Laureate) kept the worldwide debate alive through a number of subsequent books and publications, amongst which **The Intelligent Individual and Society** (McMillan, New York 1938), **The**

"Effective languages" are those which allow scientists and technicians to *describe*, *design*, *predict* ad *produce* events with the *desired* precision, in order to obtain the expected results systematically. In modern science and technology, words, concepts and symbols represent *specific operations* of identification, measurement, quantification and calculation of physical things, effects, and events which are observable *to the extent to which* they are identifiable and *measurable* by use of conventional specific instruments.

In electro-magnetism, for example, if one speaks of "electric potential", or of "magnetic field", means *two particular quantities* which can be identified and measured in determined circumstances by use of special instruments designed and constructed for those specific purposes.

In chemistry, writing $2H+O=H_2O$ means the possibility of obtaining *one molecule of water by combination of two molecules of hydrogen with one molecule of oxygen*; "molecules", "water", "hydrogen" and "oxygen" being "things" that are identifiable and quantifiable through conventional specific operational procedures and instruments. And so on.

Compare, now, the "control" allowed by the *scientific concepts* recalled above with the "control" allowed by *non-operational concepts* such as, for example, the concepts of "complex", "super-ego" and "unconscious" in psycho-analysis, or "elasticity of the demand", "propensity to consume" and "market transparency" in economics, or "divine providence," or "The Truth", or "guilt" in religions.

In principle, the specific connection of the *operational* language with any described event allows anyone to *replicate* and *verify* – at will and with no ambiguity – the *objective* meaning of statements made and texts written and diffused in modern *scientific* language. Out of the sphere of *operational languages* there is no modern science, but only abstract speculation and philosophy, which – as to *practical* effectiveness – no mathematical language can turn into *scientific stuff*; unless mathematics itself is the stuff addressed.

Obviously, this is not to say that *non-operational* words or concepts have no effects! The point is that these other effects (*i.e.*, the effects that may be caused-by or associated-with words and concepts of common use or religions or metaphysics or any abstract theories) cannot be kept *under* objective, precisely predictable, replicable, constant, *measurable control*.[11]

1.3 *Evolving systems*

Biological organisms are the best examples of complex evolving systems. Though every organism is characterized by distinguishing features, which – in a way – make each living individual "unique", contemporary biological sciences have recognized and understood most of the physiological systems which are common to the various living species and govern their functions and evolution. Biological sciences, from biophysics and biogenetics to molecular biology, from botanic and zoology to agronomy, veterinary, medicine and related ones, can identify, analyse and – to a remarkable extent – control basic biological structures and functions that work as *typical systems*, which are actually identical in biological *types* of each species, family, etc.

The knowledge achieved by contemporary biological disciplines allows specialists to replicate their practice in biological experiences indefinitely, with an increasing high probability of success

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Operational Aspect of Meaning (in *Syntheses* magazine, vol.VIII, Nos. 6-7, 1950-1951), **Science and Common Sense** (in *The Scientific Monthly*, vol.74, no.1, 1954).
[11] Operational languages include any *standardized* technological language, such as sets of detailed plans, drawings, models and specifications prepared to direct and supervise constructions, experiments or events of any kind. As to the technological sphere, computational codes are perfect examples of operational language. Out of the sphere of modern positive science and technology, the best example of "operational language" is probably given by written classical music.

in pursuing their purposes. Such an "operational" possibility of replicating experimental successes explains why biological disciplines are included in the set of modern sciences.

Instead, the systems that are usually addressed in essays like this of mine are those systems whose behaviour and evolution have a connotation of uniqueness and normally escape commonly shared understanding and control techniques. Systems of the kind are, to mention a *few significant examples only*, social communities, special institutional groups, or groups of interconnected economic activities and organisations.

So far, as historical memory proves, economics, politics, sociology, psychology and a number of related disciplines could not show any "scientific" success in understanding and controlling systems of that kind, just because such systems cannot be *typified*. It is the repeatedly frustrated and unceasing trouble suffered by high stature scholars of recent centuries, while some of them are nevertheless still reluctant to renounce attempts to bring the "complexity" of systems like the mentioned ones under a "scientific" control.

Once made it clear that the theory expounded in this paper is not a "scientific" theory, but only the description of a way of reasoning, I can afford to explain what is meant for "system" in this essay.

In the theory that follows, "system" means any *mental representation* of a set of *recognized* and measurable *varying* interactions between the respective "sources" and "destinations" that "compose" the system.

The *recognition* of any such system depends on the observer's specific interest and culture, and consists in the selection of particular events, according to the observer's attention and linguistic structure. For the observer, the kind of system addressed here is characterized by the following basic aspects:

(a) the *intrinsic variability of the interactions* recognized and measured: The interactions constitute the system's "**variable elements**". The "interactions" may be *flows of any measurable thing* (to be clearly *specified* case by case), which are generated or absorbed by *sources* and *destinations*, respectively, during any conventional unit of time;

(b) a "**source**" is *not necessarily* also a destination of interactions, and – *vice-versa* – not every "**destination**" is *necessarily* also a source of interactions, though – in general – the theory considers every "**component**" of the system as both origin and destination of interactions. The identified sources/destinations are finite in number and form the "**internal components**" of the system;

(c) one special **non-identified** component of the system, which is both "source" and "destination" of interactions with part or all of the other components, is *always* associated with the set of the system's internal components. Such *unidentified* source *and* destination of interactions is referred to as the "**external component**" of the system (it may be considered as the *rest of the universe* with respect to the studied system);

(d) any "source" may also be the destination of part (or all) of the interactions it generates: That part of interactions is referred to as the "**self-interaction**" of the source. There is to note, in particular, that the *specific* self-interaction of the *external componen*t is a systematic unknown, which cannot be detected directly, but only calculated – case by case – on the basis of all the other interactions of the system, which are measurable by assumption;

(e) the system *evolves* through changes in the activity of its *components*; these will *also* be referred to as the "**activators**" of the system. The system's activity is expressed by the distribution of its *elements* (the interactions) between its *components* (the activators).

The variability of the interaction distribution is thought of as prevailingly due to the observer's perception that most interactions do not occur randomly, but are generated and bound for the respective destinations in view of *ends to achieve*. This is also a way to mean that the existence, nature and activity of each source is thought of as depending on the existence and activity of the other components of the system; while one cannot exclude the presence in the system of randomly generated or mislead interactions.

In this connection it is also perceived that the system's *activators* (*i.e.,* the system's components) can *modify* the respective actions under the rise of unpredicted *constraints* that might improve or worsen the effectiveness of their activities. In this sense, the thought system is both *evolving* and *adaptive*, since the occurrence of alterations in the generation and distribution of the interactions brings about *transformations* but does not necessarily imply the disappearance of the system, albeit such an event is in principle possible at any stage of the system's evolution.

Therefore, starting from any detected initial configuration of the interaction distribution, changes in the system shall be expected that imply both re-distributions of the interactions and modifications in the *action potentials* of the system's components.

As shown in the chapters that follow, a salient feature of such a *thought system* is that all the inherent characteristic quantities (parameters, constants, indicators, etc., as defined by the theory), can be expressed in function of the recognized and quantified interactions *only*. This means also that, once the system's specific interactions are quantified by appropriate methods and instruments, all of the significant analytical conclusions are also quantified and expressed independently of any other quantity that is extraneous to the system of the interactions described and represented.

The *properties* of the system I have just introduced hint clearly at the representation of systems whose components direct the respective actions according to "**intents**" (the *ends to achieve* I have mentioned above), which are pursued by each *activator* through interactions with other *activators* that recognise each other as such.

Thus, the point to account for is that the *activators* (*i.e.,* the system's components) can in general express a certain degree of *intelligent* (thoughtful albeit not necessarily *brilliant*) [12] activity. As we will see, the involved overall *degree of intelligence* could be measured by the *degree of effectiveness* expressed by the system's activity, according to the level of the system's internal organization that I dub "**syntropy**".

The complexity of the system is strictly related to the intrinsic impossibility to predict *stable* – even if minimal – changes in the *initial* interaction distribution. Temporary accidental or periodical changes (in the interaction pattern of *intentional* interactions) can usually be observed or thought of as effects of seasonal or recurrent modification in the overall (natural or cultural) context. Evolution processes, however, are caused by *irreversible changes* in the *set of intents* that constitute the actual structure of such systems, as it is also denounced by irreversible changes in the initial (basic) interaction distribution.

The *intents* form the dynamical structure that determines the interactions between activators. Any evolution of the *intents* implies a corresponding evolution of the whole system. (As previously promised, "intents", "structure" and the rest will carefully be defined by the theory in function of the interaction distributions).

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[12] I am taking this *smart* definition from **Complex Adaptive Systems**, by J. E. Miller & S. E. Page, Princeton University Press, 2007, p.3, *op. cit.*

1.4 *Systems of intentional interactions*

The fundamental objects of this theory are the measurable flows of "intentional interactions" between the components of any represented system.

The substantial image of any described system does not differ from that of an interaction distribution seized or thought of as occurring between any kind of system components during any conventional time unit.

Stating that the interaction flows are *intentional* is positing that a quantifiable "intent" is always associated with any interaction unit per time unit. Thus, any interaction shall be viewed as some conceptual connection of a *flow of measurable things* with the *expression* of the intent (the end to achieve) that *motivates* the interaction.

In a very coarse similitude, I deem it useful to compare the concept of "intentional interaction" with the concept of "force" in classical mechanics. "Force", in physics, is currently understood as a measurable quantity resulting from the mathematical product of a material "mass" with the acceleration it undergoes case by case. Actually, "mass" is a constant property inherent in any *physical thing*, but it is not a quantity measurable in itself. The "mass" of any material thing is a quantity that can only be assessed indirectly as the *ratio* of the measurable force applied (such as *the weight* of the thing, for example) to the acceleration the thing undergoes, which - in the case of the weight (the force applied) – is the gravity acceleration. Thus, while the applied force and the relevant acceleration are directly measurable, the quantity of "mass" involved can only be determined through a mathematical operation made on the measurements of the two other quantities. Nevertheless, "mass" is always and intrinsically associated with "force".

"Interactions" are *flows of anything real* (commodities, people, information, energy, etc.) from any *activator* to another one of the system. "**Flows**", as *transfer per time unit of real things* from any source to any destination, are by assumption always measurable; otherwise it's a system that does not regard this theory. The system's interaction flows may consist of either homogeneous or inhomogeneous quantities: in the latter case, however, it is necessary to establish an adequate quantification criterion, which allows the system's description to express the detected flows in homogeneous measurement units.

The "intent" - as a quantity indissolubly tied to the respective "interaction"- shall instead be introduced through a rigorous definition, as can be provided by an appropriate mathematical formula. (Here is the rough analogy with the definition of "mass" in mechanics).

Actually, the whole theory rests on the suggested solution to this problem.

The solution formula achieved for the "intent" expresses a specific logical relationship between each interaction flow and the specific related set of other flows.

The formulation of the problem and its solution come from the assumption that any specific system of *interactions* conveys the information that is necessary to obtain from the system observed and represented.

The interaction system being postulated as consisting of *measurable elements* makes the theory possible of *operational* application and check, albeit the applications of this theory are severely conditioned by the methods used in quantifying interactions through direct surveys.

It is worth stressing again that the "interactions" addressed by this theory are always meant as *flows*, *i.e.,* as *quantities* that start moving from sources to destinations *in a given time unit*. For instance, if it is "flows of commodities" or "flows of people", the interactions consist of the **transfers** of commodities or people, respectively, which are **generated during any conventional unit (or period) of time**.

Therefore, the "physical dimension" of an "interaction flow" is *systematically* given by the ratio of a *specific quantity* to the *time* in which the quantity's transfer occurs. However, almost all of the subsequent theoretical discussion does not mention the "time" at denominator of the ratios that define the "flows", since it is considered as obviously implied.

 In particular, dealing with "interaction probabilities"- as defined by ratios of flow to flow – the relevant time unit is elided along with the physical dimension of the probabilities. Nevertheless, it remains understood that the system of *interaction probabilities* relates to the flow distribution that is detected or thought of as occurring during the specified period of time.

2. Representation of the system

2.0 *The system's size and basic symbolism*

This chapter introduces additional definitions in order to establish the terminology and the symbolism proper to the theory. As expected, most definitions adopt mathematical symbols and expressions, whose basic function is to minimize language ambiguities.

In this connection, it is soon necessary to remark that a number of terms such as "effectiveness", "costs", "investments", "structure", "potential", "disorder", "order", and several other ones, which will appear in this paper, shall be understood according to the meaning that pertains to the relevant context only. The concepts recalled by those terms shall not be referred to any other use made by other theories in other contexts, since every *specific* concept formulated in this theory undergoes a *circumstantial re-foundation* of its meaning, still with a view to minimizing ambiguity and misunderstanding.

Any represented system will in general consist of *N* components, including the *external component* defined in the previous chapter. Therefore, *N*-1 is the number of the *internal components*, which form the "main system", *i.e.*, the set of well identified components.

Number *N* will be referred to as the "size" of the system.

The *elements* of the system are the interaction flows, which have been defined as *transfers per time unit*. For identification purposes, these elements are represented by letter *"T"* labelled by couples of index numbers, such as *"0, 1, 2,…, n"*, which , during the analysis that follows, are currently replaced by generic literal symbols like *j* and *k*. These *labels* are the "names" of the system's components. Thus, for example, symbol $T_{3,26}$ represents the quantity of transfer generated in any conventional time unit by component *"3"* and bound for component *"26"*, as – generically – T_{jk} represents the quantity of transfer generated by component *j* and bound for component *k*. The analysis treats the system's elements *as if* they were homogeneous quantities.

In general, it is assumed that – in the time unit considered – the transfer T_{jk} from component *j* toward component *k* is different from the transfer T_{kj} from component *k* toward component *j*, with the only exception of the *self-interactions*, *i.e.*, the transfers whose destinations are the same components that generate them. Thus, we can in general write that:

[2.1] $T_{jk} \neq T_{kj}$; $(\forall j \neq k ;\ j, k = 0, 1, 2, …, n)$,

which means that inequality [2.1] is true of any pair of components *j* and *k* except when the two components coincide, that is when both the generated and received transfers coincide in one and the same flow.

Any *transfer* T_{jk} (whatever indexes *j* and *k*) may equivalently be dubbed "oriented transfer" or "oriented flow" or "oriented interaction".

Therefore, the whole interaction system can be represented by a square matrix of oriented transfers, as follows:

[2.2]
$$\left\| \begin{array}{ccccc} T_{00} & T_{01} & T_{02} & ... & T_{0n} \\ T_{10} & T_{11} & T_{12} & ... & T_{1n} \\ T_{20} & T_{21} & T_{22} & ... & T_{2n} \\ ... & ... & ... & ... & ... \\ T_{n0} & T_{n1} & T_{n2} & ... & T_{nn} \end{array} \right\|$$

This square matrix consists of $(n+1) \times (n+1) = (n+1)^2$ elements. This is also the number of all the interactions detectable in the system.

The index "*0*" refers to the *external component* (*i.e.,* to the "rest of the universe").

The elements of each row of the matrix represent the transfers from the respective source-component to every destination-component of the system, including the source itself. The elements having couples of identical indexes in the principal diagonal of this matrix represent the system's *self-interactions*.

The elements of each column of the matrix represent the system's transfers bound for the same destination-component, as this is indicated by the second index of the figure: it is the set of transfers expected on arrival at each destination-component.

For further use, consider *the size N of the system* as also defined by

[2.3]
$$N = \sqrt{(n+1)^2} = n+1$$

In this connection, it is worth considering that the "main system" defined above is represented by the square matrix of $n \times n = n^2$ elements that are *not* marked by index "*0*".

The matrix [2.2] may be referred to as "interaction matrix", or "transfer matrix" or "activity matrix", indifferently.

2.1 *The system's configuration*

In representing the system according to the configuration that the observer (or the analyst) perceives as the system's "original state", the matrix [2.2] shall conveniently be supplemented by the representation of the data concerning three sets of relevant *total values*, which – as will be shown in subsequent sections of this paper – play major significant roles in developing the theory. Therefore, the matrix [2.2] is completed as follows:

[2.4]

T_{00}	T_{01}	T_{02}	...	T_{0n}	O_0
T_{10}	T_{11}	T_{12}	...	T_{1n}	O_1
T_{20}	T_{21}	T_{22}	...	T_{2n}	O_2
...	...	...	...	...	...
T_{n0}	T_{n1}	T_{n2}	...	T_{nn}	O_n
D_0	D_1	D_2	...	D_n	T

In this new matrix, the column of symbols O_j , (*j = 0, 1,..., n*), represents the sums of the figures of the respective rows; *i.e.,* each symbol represents the *total transfer* originated from the same source-component; whereas the symbols D_k , (*k = 0, 1,..., n*), in the lowest row of the matrix

represent the sums of the respective columns, *i.e.,* each symbol represents the *total transfer* that in the system is bound for the same destination-component.

Symbol *T* represents the sum of the whole set of the system's interactions occurring in the time unit (or period) considered. Table [2.4] shows that the sum *T* of all the transfers O_j generated in the system during the period of time considered is also the sum of all the transfers D_k expected on arrival at the system's destinations during the same period of time.

All this is synthetically summarized in mathematical terms by the following *definitions*:

[2.5] $$\sum_{k=0}^{n} T_{jk} = O_j \; ; \qquad\qquad (\forall j)\ ^{13}$$

[2.6] $$\sum_{j=0}^{n} T_{jk} = D_k \; ; \qquad\qquad (\forall k)$$

[2.7] $$\sum_{j=0}^{n} O_j = \sum_{k=0}^{n} D_k = \sum_{k=0}^{n}\sum_{j=0}^{n} T_{jk} = T \; .$$

As to the notation, however, I deem it convenient, from now on, to simplify the very frequent use of the symbol "Σ" for "sum" as follows:

[2.8] $$\sum_{j=0}^{n} = \sum_{j} = \sum \;\; ; \;\; \sum_{k=0}^{n} = \sum_{k} = \sum \;\; ; \;\; \sum_{j=0}^{n}\sum_{k=0}^{n} = \sum_{j,k} \;\; ,$$

because there is *no risk of ambiguity when all the indexes specified by this symbol and/or by the addends are involved* in the sum operations.

Furthermore, it is often convenient to express any matrix by indication of its generic element between brace-brackets: for example, $\{T_{jk}\}$ to express the matrix [2.1]; or $\{O_j\}$ to express the column-matrix of the total transfer generations included in the matrix [2.4], and $\{D_k\}$ to express the row-matrix of the total transfer destinations in the same matrix.

It is also possible to express the interaction matrix relative to the "main system" alone by use of the symbol $\{T_{jk \neq 0}\}$, which is the matrix of the interactions occurring between the system's "internal components" only.

The use of brace-brackets regards any kind of matrices, as specified according to necessity.

With regard to the **terminology**, the matrix [2.4] (or – equivalently – the set of matrices $\{T_{jk}\}$, $\{O_j\}$, $\{D_k\}$ together) will be referred to as "**the system's configuration**".

The set of the two matrices $\{O_j\}$ and $\{D_k\}$ will be denominated "**the base of the system**", while – separately – the matrix $\{O_j\}$ forms "**the system's *origin semi-base***", and the matrix $\{D_k\}$ forms "**the system's *destination semi-base***".

As a side consideration, note that infinite different configurations exist for any given identical base, whereas once given the matrix $\{T_{jk}\}$ the relevant *base* (*i.e.,* $\{O_j\}$ and $\{D_k\}$) remains univocally fixed. (In other words, an infinite number of *different* activity matrices $\{T_{jk}\}$ of the same size may all have an identical *base*).

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[13] To remind the reader that symbol " $\forall$ " associated with any index means that the regarded relation is true irrespective of the "value" of the index. Therefore, in this case – as well as in analogous cases – it means that the definition [2.5] holds true for every component of the system.

2.2 *Measurement of the interactions & interaction probabilities*

It is supposed that the necessary surveys are carried out when – *according to the observer's perception* – the interaction system is in its "normal condition"; which means that the system appears to the analyst in a state of *temporary* or *relative equilibrium* with respect to the overall dynamics of the processes to be represented. Dealing with *intentional interactions*, the *relatively stable state* thought of during the observation period is supposed to last enough to allow the interactions to start producing the effects expected by the system's activators.

In practical applications, the possibility of compiling an adequate interaction matrix depends on the complexity of the system to survey in conjunction with the available financial resources and the accuracy of the survey methods and techniques adopted.

With rare exceptions, the methods and techniques shall usually respond to statistical criteria and focus on appropriate *samples* of the interaction flows to detect and assess.

Moreover, in most cases, the survey findings must eventually be expressed by homogeneous figures, which in turn depend on the different homogenization methods that are used in accordance with the different specific applications of the theory.

The adoption of statistical criteria is not only a practical necessity against the *instant variability* of the interactions detected during the survey times, but also a theoretical principle to avoid accounting for surveyed accidental details that may jeopardize the reliability of the system's representation. Not to forget that the system is by assumption evolving, and that the system's image represented by the survey data is only the image of a momentary *conventional* stability state. The system's "**original configurations**" - as *temporarily stable states* addressed by the theory - are only convenient *assumptions*, which are necessary for developing a logical representation of the evolution processes described.

Normally, the description of evolution processes, both in this theory and in other ones, consists in the representation of at least two kinds of "configurations": one relative to *equilibrium states* (be these temporary or permanent conditions of the system); the other one relative to *transition states*, which are represented in *sequences* of unstable configurations.

The preceding considerations suggest that – in all cases – any adequate assessment of an interaction system consists of the *most probable measurements* of the transfer flows surveyed. This is the reasonable conclusion that leads to turn any *interaction matrix* into a system of *interaction probabilities*, which bears a number of important theoretical and practical simplifications.

The first simplification is the elimination of most worries concerning the adequacy of the criteria followed to measure and homogenize the flows detected. Any *interaction probability* is the ratio of a flow to a flow, so that any consistent measurement criteria used to detect and record the transfer flows does not change the value of the ratio.

A second major simplification, particularly appreciated in technical data processing, is just the possibility of dealing with *numbers* void of physical dimension.

The third principal virtue of any set of *probabilities* is the possibility of processing them by means of logical instruments proper to the theory of probability.

Therefore, once detected and recorded the interactions of the system to represent, it is also possible to compile the respective matrix of *interaction probabilities* that verify the following equations:

[2.9]
$$\sum_k P_{jk} = P_j ; \qquad\qquad (\forall j)$$

[2.10]
$$\sum_j P_{jk} = Q_k ; \qquad\qquad (\forall k)$$

[2.11]
$$\sum P_j = \sum Q_k = \sum_{j,k} P_{kj} = 1 .$$

This set of equations is simply resulting from the division of the definitions [2.5], [2.6], [2,7] by the total transfer T detected in the system. Therefore:

[2.12]
$$P_{jk} = \frac{T_{jk}}{T} \qquad\qquad (\forall j, k)$$

expresses the probability (in the given period of time) that a transfer flow is generated by component j and bound for component k; while

[2.13]
$$P_j = \frac{O_j}{T} \qquad\qquad (\forall j)$$

expresses the probability (in the given period of time) that any transfer in the system is generated by source j ; and

[2.14]
$$Q_k = \frac{D_k}{T} \qquad\qquad (\forall k)$$

expresses the probability (in the given period of time) that any transfer in the system is bound for component k.

It is worth noting that the relations [2.11] – and obviously those from [2.12] to [2.14] - express three different *discrete probability distributions*.

A *probability distribution* exists when the sum of all the probabilities of a given set is equal to 1. (In this connection it is important to remind the reader that any probability value may only range between 0 and 1).

2.2.1 *What interaction probability means*

Since centuries theorists debate about the *meaning* of "probability", without a final conclusion though. Dictionaries rest on the common sense and define probability according to its simplest mathematical meaning, *i.e.*, "probability" as a "ratio" showing the chances that a particular event will occur.

As to the interactions flows detected during any on-the-field survey, the events are just occurring and measured, albeit by methods that cannot ascertain the maximum precision of the figures recorded. In this connection, it must be remarked that no absolute certainty characterizes any

laboratory measurement operation; this is the fact that has led experimenters to lay the foundation of the "theory of measurement", which concerns all modern scientific and technological activities.

In the context of the theory expounded in this paper, I deem that the concept of "probability" may conveniently express the *degree of reliability* of the measurements effected, accounting for the analyst's *subjective knowledge and information* about the system he tries to describe. It is a standpoint close to that of the theorists of the *operational subjective* conception of "probability", who remark that – in practice – any probability assessment depends on what one knows about any *possible* event.[14] The operational subjectivity of the probability assessment as "*opinion* based on the information one possesses" is discussed also in relation to the so called "objective possibility of occurrence" of the events considered, as in the cases of dicing or lotteries.[15]

As shown ahead, operational subjectivity, information and uncertainty play a joint fundamental role in the construction of the theory expounded in this essay.

2.3 *Expectations associated with the interactions: Efficacy and   randomness of the transfers*

Assuming that the interaction flows are "intentional" implies the assumption that the *activators* of the flows attach the expectation of some effect to the interactions they generate. For instance, if the interaction flows consist of transfers of commodities between producers and buyers, one may assume that the producers expect a payment from the buyers. In general, however, it is not necessary to think of the flows' efficacy as of the benefits of a successful sale, or as of the benefits of any commercial, financial or any other business transaction, unless such transactions are considered as included in a very large unspecified variety of possibilities. Think, for example, of the flows of people moving from place to place in any urban or regional area, or of the exchanges of information flows through a postal or telephone network. Such kinds of flows aim at *ends* that are difficult – or impossible – to define and quantify either in conventional economic terms or in terms of achieved efficacy.

There is no inconvenience in defining the "**efficacy**" associated with any interaction flow as the mathematical difference between the "results" *intended* by the interaction and the adverse factors that oppose the achievement of the results. The "adverse factors" I mean here are dubbed "the costs" of the interaction: by a terminological convention, "costs" are all kinds of burdens, constraints, hindrances, frictions, errors, etc., which can attenuate or cancel or even overcome the "value" of the achieved "results".

Both "results" and "costs" shall be thought of as entities somehow quantifiable by homogeneous measurement units, *albeit the use of those concepts in this theory does not presuppose any direct measurement of the relevant quantities.*

In principle, the observer/analyst assumes that the values for "results" and "costs" are subjectively assessed by the system's activators, according to evaluation criteria that are likely different from case to case and *objectively unidentifiable.* The analysis that follows will represent and process "efficacy", "results" and "costs" associated with the interaction flows symbolically, according to the meaning expressed by the following mathematical relation

[2.15] $$U_{jk} = B_{jk} - C_{jk}\ ; \qquad\qquad (\forall\, j,k),$$

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[14] Italian mathematician Bruno de Finetti (1906-1985) was one of the leaders of this school of thought: ***Teoria delle probabilità,*** (2 volumes), Einaudi, Torino 1970. (English version, *Theory of Probability,* 2 vol., Wiley, New York, 1974).

[15] As a quick reference, see also the discussion on the topic I have summarized in the article **Syntropy, *Definition and Use*** published by this magazine in December 2008, Paragraphs 4 & 5.

in which U_{jk} represents the "efficacy" associated with any interaction flow T_{jk}, while B_{jk} and C_{jk} represent the relevant "results" and "costs", respectively. [16]

After division of the equation [2.15] by T_{jk}, a **mean efficacy** is symbolically expressed by $u_{jk} = U_{jk}/T_{jk}$, $(\forall j,k)$, which shall be *associated with every interaction unit* of flow T_{jk}; whence the definition

$$[2.16] \qquad\qquad (u_{jk} = b_{jk} - c_{jk})T_{jk} = U_{jk}\,; \qquad\qquad (\forall j,k),$$

in which b_{jk} and c_{jk} symbolize the *mean result* and the *mean cost*, respectively, associated with any unit of the flow T_{jk}.

The theory formulates the concept of "intent associated with every interaction flow" as a simple mathematical quantity directly proportional to the unit-flow's *mean efficacy u_{jk}* defined above. The definition of the flow's *intent*, as obtained through the analysis that will follow, is given by:

$$[2.17] \qquad\qquad \mu_{jk} = \lambda u_{jk}\,, \qquad\qquad (\forall j,k),$$

where μ_{jk} is the "intent" of the respective interaction flow and λ is a system constant that depends on the measurement system used for quantifying the interaction flows.

Subsequently, as announced in Paragraph 1.3 of Chapter 1, the "intents" of the interaction flow system will be quantified precisely and *only in function of the transfer flows detected*, in compliance with one of the chief tasks of the theory.

In this context "intent" as well as "efficacy" do not *necessarily* relate to *positive quantities*. In particular, an interaction is referred to as "random interaction" if its "intent" is nil (*i.e., $\mu_{jk} = u_{jk} = 0$*), since the "*end*" of the flow is in this case equated by the analyst with any flow whose results and costs are also nil, in connection with the fundamental assumption that the observer cannot access the measurement of "efficacy" and "intents" directly.

In general, as it will be explained, *the analyst considers* that the interaction system is in a state of "randomness" when the system's configuration is characterized by an interaction distribution that does not express any associated distribution of "differentiated intents", or – more precisely – when *all* the intents expressed by the system may be supposed to be equivalent to each other (*i.e., $\mu_{jk} = \mu^{*}$*, $\forall j,k$).

Instead, as per further discussion, *negative values* appearing among differentiated intents may be interpreted in various ways, according to the possible uses of the theory when applied to a number of particular cases.

In this theory the "intents" μ_{jk} defined above constitute the "structure" of the system, both in the form of the definitions [2.17] or – indifferently – in their exponential form given by

$$[2.17a] \qquad\qquad \varepsilon_{jk} = e^{\lambda u_{jk}}\,, \qquad\qquad (\forall j,k),$$

which will later be introduced as the system's "**relation coefficients**".

A "matrix of intent $\{\mu_{jk}\}$" – as well as the corresponding "relation matrix $\{\varepsilon_{jk}\}$" – is univocally associated with any given *interaction matrix* $\{T_{jk}\}$ and with the corresponding *probability matrix* $\{P_{jk}\}$.

The matrix $\{\mu_{jk}\}$ (or, indifferently, the matrix $\{\varepsilon_{jk}\}$) represents the *structure of the system*.

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[16] If the theory is used to describe socio-economic or financial systems, symbols U_{jk}, B_{jk}, and C_{jk} can suitably represent "gains/losses", "benefits" and "costs", respectively, as usual.

Note that the *matrix of intent* $\{\mu_{jk}\}$ may include negative elements in consequence of negative values of the relevant *interaction efficacy,* by which the intents are determined (see the definition [2.17]); whereas this is not true of the *relation matrix* $\{\varepsilon_{jk}\}$, in which all elements are exponentials, whose values are always positive (or nil *only if* the "intents" are infinitely negative). The matrices $\{\mu_{jk}\}$ and $\{\varepsilon_{jk}\}$ are different forms for representing the system's structure during the analysis, according to occasional convenience.

2.4 *Imaging the system*

In the preceding paragraphs, the principal terms used by the theory have been introduced to describe the system, as per the following summary list:

(1) the interaction flows T_{jk} (and respective probabilities P_{jk}) between the system's *components (*or *activators*), as basic *elements* that determine the system's "flow matrix" or "transfer matrix" or "activity matrix" expressed by the matrix $\{T_{jk}\}$, (*i.e.,* the matrix [2.2]) or – indifferently – by the corresponding probability matrix $\{P_{jk}\}$;

(2) the *base of the system*, as expressed by the set of *origin semi-base matrix* $\{O_j\}$ and *destination semi-base matrix* $\{D_k\}$ (or – indifferently – by the set of the two respective probability matrices $\{P_j\}$ and $\{Q_k\}$);

(3) *the system's configuration*, as expressed by the *activity matrix* $\{T_{jk}\}$ together with *the system's base* $\{O_j\}$ and $\{D_k\}$; (or – indifferently – as expressed by the set of the three respective probability matrices $\{P_{jk}\}$, $\{P_j\}$ and $\{Q_k\}$);

(4) the generic *mean efficacy* u_{jk} associated with any *interaction unit*, expressed by the difference between the *mean result* b_{jk} achieved by the same interaction and the relevant *mean cost* c_{jk} involved;

(5) the generic *intent* μ_{jk} of any *interaction flow* T_{jk} , as *motivation* of the flow, which is a linear function of the flow's mean efficacy;

(6) the *intent matrix* $\{\mu_{jk}\}$ as *structure* of the system; occasionally, the same structure may also be represented by the *relation matrix* $\{\varepsilon_{jk}\}$, whose elements are the exponentials of the elements of the matrix $\{\mu_{jk}\}$.

By assumption, the elements of the transfer matrix $\{T_{jk}\}$ are the only quantities that must be considered as directly measurable (through on-the-field measurement operations), whereas all the remaining quantities defined above shall be expressed or determined indirectly, in function of the values assessed for the elements of the matrix $\{T_{jk}\}$.

2.5 *Disorder and order in the system's image. Entropy and Syntropy*

The observer gets an image of the system that he can recognizes as such through the perception of a certain degree of *order* established between the system's components and interactions, according to all that which he knows or guesses about the events observed. His perception is connected with the observation that a major part of the interactions that constitute the system's

activity are not randomly generated and distributed amongst the system's components, but show instead criteria of destination selection that reflect in the differentiated intensities of both production and confluence of the detectable interaction flows.

In other words, to the observer mind the system's components show a sort of hierarchy as to their *capacity* to produce, convey and absorb interactions. Should this not be true, the observer would have no reason for thinking that the events under his attention may constitute a "system".

One operational means for assessing the degree of *disorder* in the studied system is provided by the concept of *statistical uncertainty*. As known, the concept was precisely formulated in probabilistic terms by USA mathematicians Shannon and Weaver in dealing with the theory of information transmission and processing.[17] Shannon proved the existence and uniqueness of a mathematical quantity, to be appropriately labelled "statistical uncertainty", which is always associated with any distribution of probabilities. Given the *probability distribution p_i*, ($i = 1, 2,…,$ n), of any set of n different probabilities with which the observed events may occur, (remember that – by definition – it is $\sum_{i=1}^{n} p_i = 1$), there is a quantity E, dubbed "statistical uncertainty", which is univocally associated with the distribution of p_i and expressed by

[2.18]
$$E = -\alpha \sum_{i=1}^{n} p_i \log p_i \quad ,$$

in which α is a numerical constant that depends on the base of the logarithms "log".

Shannon named uncertainty E "**entropy**", since both shape and conceptual meaning of the equation [2.18] corresponds to the mathematical definition of "thermodynamic entropy" introduced by the statistical mechanics formulated by Ludwig Boltzmann in years 1877-1880.[18]

Quantity E is always a *positive* quantity: Consider that probabilities are *positive* quantities, generally less than 1, and that *the logarithms of positive numbers less than* 1 *are negative numbers*. Consider also the following particular cases:

if the probability distribution consists of one probability only expressed by $p=1$, then $\log 1 = 0$ and uncertainty $E = 0$. This case regards *one only* possible event, whose occurrence is certain;

if $p_i = 0$, whatever p_i, then $\lim_{p_i \to 0} (p_i \log p_i) = 0$; therefore, if all $p_i = 0$ then uncertainty $E = 0$ too: It means there is no uncertainty because no event is possible;

if, in the observer's opinion, *all* of the observed events have the same probability of occurrence, as it is expressed by $p_i = p = 1/n$, $\forall i$, (n being the number of the possible events), then

[2.19]
$$E = -\alpha \sum_{1}^{n} \frac{1}{n} \log(\frac{1}{n}) = -\alpha n(-\frac{\log n}{n}) = \alpha \log n ,$$

---

[17] Claude Shannon (1916-2001) & Warren Weaver (1894-1978), *The Mathematical Theory of Communication,* University of Illinois Press, Urbana, 1949.
[18] Ludwig Boltzmann (1844-1906), Austrian physicists, defined equation the [2.18] for thermodynamic entropy in formulating his *kinetic theory of gases*. The equation establishes the degree of *molecular disorder* in any volume of gas in a given thermal state, p_i being the *microstate probability* of each molecule in relation to its temperature and pressure.

which value represents the **full uncertainty** that the analyst associates with the uniform distribution of probabilities.

Both the formula [2.18] and relevant concept may conveniently be translated for the analysis of the probability distribution $\{P_{jk}\}$ introduced by [2.9] to [2.13], after adopting natural logarithms "ln" and assuming conventionally $\alpha =1$ for the sake of simplicity, to obtain

[2.20]
$$E = -\sum_{k=0}^{n+1}\sum_{j=0}^{n+1} P_{jk}\ln P_{jk} = -\sum_{j,k} P_{jk}\ln P_{jk} \ ,$$

which (also allowing for the simplified notation established with [2.8]) expresses the *statistical uncertainty* or the *degree of disorder* or the *entropy* that the observer can associate with the N^2 elements of the system under analysis.

In the extreme case of a transfer probability matrix whose elements are *all identical* to each other, which case can be symbolized by $N \times N$ matrix $\{P_{jk} = P^i = 1/N^2\}$, **full entropy** or **full disorder** H is ascribed to the (would-be) system and expressed by

[2.21]
$$H = -\sum_{i=1}^{N^2} P^i \ln P^i = -N^2\left[\frac{1}{N^2}\ln\left(\frac{1}{N^2}\right)\right] = 2\ln N \ .$$

Quantity $H=2\ln N$ will be referred to as "the system's **entropy potential**", which is directly proportional to the logarithm of the system's size, as defined by the number N of the system's components.

Entropy potential H expresses an *entropy extreme* that can never be achieved by the system until the observer deems the system existent; (or else, exceptionally, only if the observer/analyst considers any finite set of identified "components" as a *virtually existing* system).

Conventionally, the *full entropy* or *full disorder state* of a system is referred to as the system's "**amorphous configuration**", which is only mentioned as a *border* as well as *unattainable* state, since it must actually be considered as the *vanishing state* of any system.

I shall later try to explain the concept through a simple but not banal example, with a view also to evidencing the principle, introduced in the preceding chapter, that recognition and definition of any system are subjective mental operations: These are proper to the analyst/observer, according to his own culture and to the information that is available to him about the *events* on which his interest is focused.

Meanwhile, it is worth observing that the entropy (or degree of disorder) of the system can also be defined in terms of the physical quantities $\{T_{jk}\}$ that represent the system's interactions, accounting for the definitions given by the relations [2.7] and [2.12]. Therefore, entropy E may also be defined by

[2.22]
$$E = -\sum_{j,k}\frac{T_{jk}}{T}\ln\frac{T_{jk}}{T} = -\frac{1}{T}\sum_{j,k}\left[T_{jk}(\ln T_{jk} - \ln T)\right] =$$

$$= -\frac{1}{T}\left(\sum_{j,k} T_{jk}\ln T_{jk} - T\ln T\right) =$$

[2.22]
$$= E = \ln T - \frac{1}{T}\sum_{j,k} T_{jk}\ln T_{jk} \ .$$

The value for the system's *entropy* (or *disorder*) expressed in this form is therefore coincident with that expressed by the equation [2.20].

Therefore, as it should appear obvious, the expression of the system's *entropy potential H* takes the same form as that shown by the equation [2.21] also when the entropy is formulated directly as a function of the transfer flows instead of the respective probabilities. In fact, the distribution of the transfer interactions in the system's *amorphous configuration* are thought of as all identical to each other and represented by the interaction matrix $\{T_{jk}=T/N^2\}$.[19] Thus, after substitution of all the T_{jk} in the entropy equation [2.22] with T/N^2, the immediate result is again

$$H = \ln T - \frac{N^2}{T}\left[\frac{T}{N^2}\ln\left(\frac{T}{N^2}\right)\right] = \ln T - \frac{N^2 T}{TN^2}\left(\ln T - 2\ln N\right) = 2\ln N \,.$$

When the system consists of a very large number of components, it might be convenient to carry out more precise calculations by use of the equation [2.22], instead of [2.20], considering that the number of elements in the interaction matrix grows with the *square* number of the components, and that all the elements of the *probability matrix* are in such cases numbers much smaller than 1. Actually, there is to consider that the first significant digits of the probability values may often appear at orders of magnitudes such as 10^{-6} and beyond.

"**Syntropy**" is then the term introduced by this essay to indicate a possible measurement of the *degree of order* that can be detected in a system, according to the system's interactions described and quantified by the analyst.

To the extent to which the observer has identified a set of *components* that interacts intentionally, it is quite reasonable to induce that the identified system is not in a state of full disorder. This implies that the level of the system's entropy E must be lower than that of the amorphous configuration expressed by H.

On the basis of this simple consideration, it may conventionally be assumed that the difference between the *theoretical full entropy H* and the *actual entropy E* associated with the system's interactions provides a significant measurement of the *degree of order*, dubbed "syntropy", inherent in the system.

Therefore, the definition of "syntropy", symbolized by S, is as follows:

[2.23] $S = H - E \,.$

This definition makes *entropy* and *syntropy* quantities complementary to the *entropy potential H*, which is a constant quantity proper to the system described; so that

[2.23a] $S + E = H,$ *constant,*

whereas S and E vary according to different possible states of the system during its evolution process.

By use of the equations [2.21] and [2.20], the definition of syntropy can also be expressed by

[2.24] $S = 2\ln N + \sum_{j,k} P_{jk}\ln P_{jk} \,,$

---

[19] Remember that $P_{jk} = T_{jk}/T,\ (\forall P_{jk})$; therefore, if $P_{jk} = 1/N^2,$ then $T_{jk} = T/N^2.$

as a function of the system's matrix of probabilities; or else by

[2.25]
$$S = \ln \frac{N^2}{T} + \frac{1}{T} \sum_{j,k} T_{jk} \ln T_{jk} \, ,$$

if expressed - by use of the equation [2.22] - as a function of the transfer matrix.

It is easily proved that both full entropy ($E = H$) and full syntropy ($S = H$) are impossible states for any *non-amorphous* system.

In this connection, consider function E as expressed by [2.22]. If an absolute maximum value were possible for this function, all the first derivatives of E with respect to each variable P_{jk} should be null, to write

[2.26]
$$\frac{\partial E}{\partial P_{jk}} = \frac{\partial}{\partial P_{jk}} \left(-\sum_{j,k} P_{jk} \ln P_{jk} \right) =$$

$$= -\left(\frac{\partial P_{jk}}{\partial P_{jk}} \ln P_{jk} + P_{jk} \frac{\partial}{\partial P_{jk}} \ln P_{jk} \right) = 0 \, ; \qquad (\forall j,k),$$

whence

[2.27]
$$\ln P_{jk} + 1 = 0 \, , \qquad (\forall j,k),$$

which is impossible, whatever P_{jk} .

(The case of *full disorder*, which defines the entropy potential H of the system, is characterized by a uniform probability distribution in which $P_{jk}=1/N^2$ is the value for every interaction probability. If this probability value replaces P_{jk} in the relation [2.27], the result is that the size of the system is *in all cases* expressed by $N = e^{1/2} = 1.64872121\ldots$, which makes no sense).

The fact is that the entropy function defined by [2.22] is subject at least to the constraint by which the probability distribution $\{P_{jk}\}$ consists of variables whose sum must always be equal to 1. Therefore, it is possible to determine the maximum value for entropy [2.22] only under the constraint established by the equation [2.11], *i.e.*, by the condition

[2.28]
$$\varphi = \sum_{j,k} P_{jk} - 1 = 0 \, .$$

The function φ of variables $\{P_{jk}\}$ is the *constraint* under which a *maximum* for entropy E can be found.

One method for the determination of the *constrained maximums* of a function is known as the *Lagrangian multipliers method*. According to this method, the coordinates of a *point of maximum* of a constrained function coincide with the coordinates of the "maximum" of another function, named "the Lagrangian of the function", which is expressed by

[2.29]
$$L = \varphi + E = \left(\sum_{j,k} P_{jk} - 1 \right) - \sum_{j,k} P_{jk} \ln P_{jk} \, ,$$

where   is a constant value that is usually referred to as *Lagrangian multiplier*. This is in turn determined by the values { $\overline{P}_{jk}$ } found as coordinates of the *constrained maximum* for *E*.

Necessary condition for *L* to have a point of *maximum* is the zeroing of its derivatives with respect to each variable P_{jk} ; that is

[2.30] $$\frac{\partial L}{\partial P_{jk}} = 0 \; ; \qquad\qquad (\forall j,k),$$

which means

[2.31] $$\frac{\partial L}{\partial P_{jk}} = \quad -1 - \ln P_{jk} = 0 \; ; \qquad\qquad (\forall j,k),$$

whence the searched solution

[2.32] $$\overline{P}_{jk} = e^{\vartheta - 1} \; ; \qquad\qquad (\forall j,k) \; .$$

Using these solutions to replace the $\{P_{jk}\}$ in the constraint [2.28], the expression becomes

[2.33] $$\varphi = N^2 e^{\vartheta - 1} - 1 = 0 ,$$
whence

[2.34] $$e^{\vartheta - 1} = \frac{1}{N^2} \quad ,$$

N^2 being the number of the probabilities $\{P_{jk}\}$; to obtain also

[2.32a] $$\{\overline{P}_{jk}\} = \left\{\frac{1}{N^2}\right\} \; ;$$

which – as seen before – expresses the probability distribution of an *amorphous system.* Because of the equation [2.21], the entropy of an amorphous system coincides with the system's entropy potential $H = 2\ln N$, so that *the amorphous system's syntropy is necessarily nil by definition* (see [2.24]); to mean that an "amorphous" system is such for *no degree of internal order* (or *organization*) can be represented for it.

The preceding notes prove also that *no absolute maximum* exists for both the entropy *E* and the syntropy *S* relevant to any *non-amorphous system*. Moreover, non-amorphous systems are subject to additional constraints that intrinsically imply diversification of the interaction probabilities. This fact – *i.e.*, the lack of uniformity in the interaction distribution – means that

[2.35] $$S = H - E > 0 \quad \text{and} \quad E = H - S > 0$$

are differences greater than zero always.

The actual meaning of these inequalities is – on the one hand – that *no system* is conceivable if it is not possible to associate any *minimal order* (or *internal organization*) with the system described. On the other hand, the same inequalities indicate that *randomness* or *disorder* can in no case be eradicated from the system completely, whatever the system described. This is in accordance with the meaning attached to the concept of *amorphous configuration* defined above, which expresses a limit *not intrinsic* to functions *S* and *E*, but only a specific border condition that makes the system non-existent.

The preceding considerations give *entropy potential H* a basic role in the system's evolution. The entropy potential does not change if the evolving system does not change its size, *i.e.,* if the system – during its evolution – does not change the number of its components, for entropy potential *H* depends solely on *N*. This fact is evidenced by the equation [2.23a], by which the constrained evolution of the system may simply be represented geometrically by an ellipse, as follows:

FIGURE 2.1

The axis of entropy potential H=2lnN
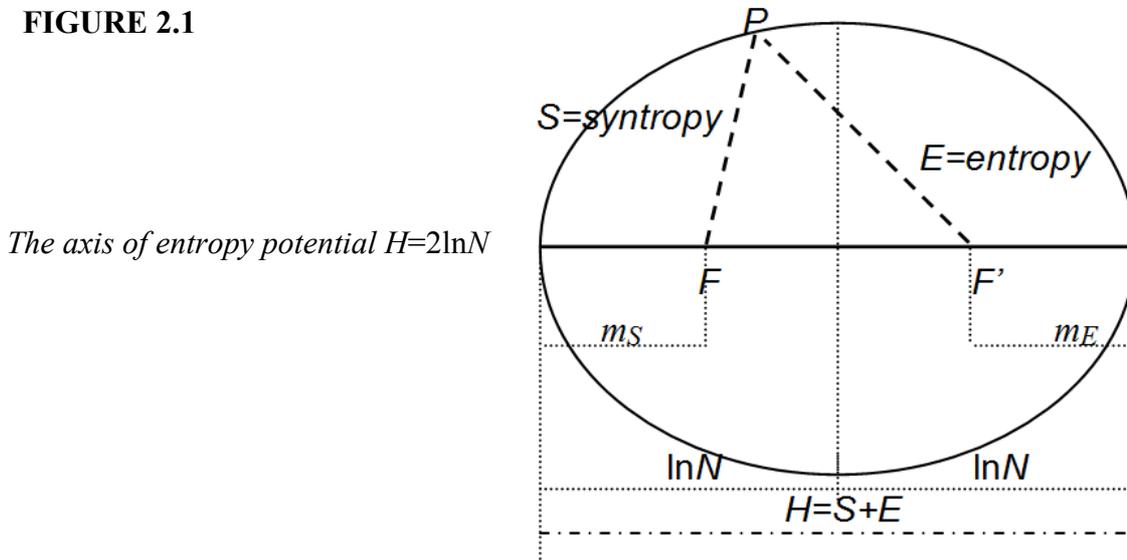
This ellipse represents one of the infinite possible elliptical "orbits" *with a common major axis,* which binds "the family" of the system's evolution paths together. Point *P* represents the *state* of the system in any given *phase* of its evolution: The *sum* of the distances $PF = S$ and $PF' = E$ from the ellipse's focuses *F* and *F'*, respectively, *is constant* and equals $H = 2\ln N$, which is the length of the ellipse's major axis; while each semi major-axis represents $H/2$. The segments m_S and m_E represent two of the infinite possible "minimum values" for *S* and for *E*, respectively, while the segments $H–m_S$ and $H–m_E$ represent the corresponding possible "maximum values" for *S* and *E*, respectively. Both "minimums" and "maximums" are jointly determined by the system's *"base configuration"*. This concept is introduced and discussed in subsequent Paragraph 2.7.

2.6 Shaping a system: A schematic example

The example that follows intends only to show – by means of a *thought-situation*[20] – how a system may be "recognized" by an "observer" through the information the observer gets about what he observes.

Suppose I come to know that, in a room of a historical palace in Rome, five representatives of five European governments meet for a couple of days to discuss confidentially major *unknown* political issues.

I plan to describe the possible *interaction system* that I expect is forming through the exchange of information occurring between the members of that group of governmental representatives.

At the very beginning I can only suppose (as it belongs to the practice of any important meeting) that each representative brings a folder with notes and documents to be consulted during the meeting. I can also suppose that each participant in the meeting may occasionally avail himself of a mobile-phone for quick consultation – as necessary – with colleagues, advisers or members of the

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[20] This *"thought-situation"* shall be taken as a conceptual equivalent of a *"thought experiment"* in physics.

respective government, to provide them with news about the meeting in progress and/or to receive possible suggestions or directives from people who stay outside the meeting room. Yet, I assume that writing personal notes or consulting documents is the *self-interaction activity* of each meeting attendant; whereas I consider the exchange of information through mobile-phones as transfers of information to and from the "external component" of the meeting system "under formation".

That beeing all that which I know initially, my unique reasonable possibility is outlining a *potential* system of information transfers, which can only be shaped like an *amorphous system* of six components, one of which is external to the "main system" of the five meeting people. Actually, given such conditions, the *information transfer matrix* relevant to the six-component group can only be based on an undetermined mean amount *x* of information uniformly exchanged between the components of the *would-be* system, according to the "amorphous " configuration here below:

[2.36]

	0	1	2	3	4	5		
0	x	x	x	x	x	x	$6x$	O_0
1	x	x	x	x	x	x	$6x$	O_1
2	x	x	x	x	x	x	$6x$	O_2
3	x	x	x	x	x	x	$6x$	O_3
4	x	x	x	x	x	x	$6x$	O_4
5	x	x	x	x	x	x	$6x$	O_5
	D_0	D_1	D_2	D_3	D_4	D_5	$36x$	T

As evident, this matrix has all the features that characterize an amorphous configuration sized *N=6*. Numbers "*0, 1, 2, 3, 4, 5*", are the "names" of the "components" of the *would-be system*, according to the observer's code; number "*0*" is also in this case used to label the unidentified "external component".

The obvious relations that can be formulated are as follows:

[2.37] $O_i = D_i$, $(\forall i)$;

[2.37a] $\sum O_i = \sum D_i = T = 36x$;

[2.37b] $P_i = \dfrac{O_i}{T} = \dfrac{6x}{36x} = \dfrac{1}{6} = \dfrac{1}{N}$; and $Q_i = \dfrac{D_i}{T} = \dfrac{6x}{36x} = \dfrac{1}{6} = \dfrac{1}{N}$; $(\forall i)$,

[2.37c] $P_{jk} = \dfrac{x}{T} = \dfrac{x}{36x} = \dfrac{1}{36} = \dfrac{1}{N^2}$; $(\forall j,k)$.

The *entropy* associated with this configuration is therefore that pertaining to an *image* of *full disorder*, proper to the *entropy potential H_6* of any (would-be) system sized *N=6*, that is:

[2.37c] $H_6 = 2\ln N = 2 \times \ln 6 = 2 \times 1.79176 = 3.58352$.

I find a way to contact the five simultaneous interpreters in their translation boxes (which are acoustically isolated from the meeting area). The interpreters follow the political discussion only through a microphone-earphones system. They can hear *online* only what people say at the meeting

when they speak at the microphones predisposed around the meeting table. Thus, they hear nothing of interpersonal conversations occurring accidentally *out-of-line* or through personal mobile-phones, nor can they seize what the attendants at the meeting write-on or read from their personal notes and documents. Besides, the interpreters are forbidden to report about the contents of the political discussion in progress.

However, the interpreters *can see* the position, attitude and gestures of the convened governmental representatives. Basing on this fact, I ask the interpreters the favour - say for "scientific purposes" – to task their box-assistants with metering and recording the time spent by *each participant*

(a) to speak to all the other participants through the microphones predisposed around the meeting table;
(b) to speak *privately*, on whatever occasion (pauses, coffee-break, etc.), with one or more other *identified* participants;
(c) both to speak and to stay silent at the mobile-phone;
(d) to write own-use-notes or reading their own documents.

Thus, using the time measured in seconds as a quantity directly proportional to the quantity of information conveyed and received (and considering such a proportionality as acceptable in a first approximation), I avail myself of significant data concerning the varied intensity of information exchanged between each pair of participants in the meeting, as per activities *(a)* and *(b)* above, together with the data concerning the interaction with the *external component,* as per activity *(c)*, and the data devoted to the *self-interaction,* as per activities *(d)*.

Therefore, the data so obtained allow me to *shape* an interaction system represented by a matrix of figures that can be processed with the purpose of assessing – for example – an approximate *weight* of the intents associated with the interaction conveyed and collected by each member of the meeting group.

The interaction system I describe responds to some interests of mine, which characterize both shape and contents of the system's configuration. Another observer, in the same situation as that I have outlined above, might either *shape* a different system or no system at all.
In other words, the system I describe is a representation of a particular real event viewed through the filter of my own mental inclination and attention.

2.7 *Base entropy, base syntropy, base configuration*

In Paragraph 2.2, after introducing the definitions [2.13] and [2.14] for the interactions' *origin probabilies* $\{P_j\}$ and *destination probabilities* $\{Q_k\}$, respectively, I have observed that each of the two sets of probabilities constitutes a *probability distribution,* for the sum of its elements equals 1. This implies that a *degree of statistical uncertainty* (or "entropy" as per Shannon's definition) may be associated with each of them: Namely,

[2.38a]
$$E_O = -\sum_{j=1}^{N} P_j \ln P_j \ .$$

dubbed "**origin semi-base entropy**", and

[2.38b]
$$E_D = -\sum_{k=1}^{N} Q_k \ln Q_k \ .$$

to be dubbed "**destination semi-base entropy**". Therefore, in general, "**base entropy** E^* " is defined by

[2.39]
$$E^* = -\sum_{j,k} \left(P_j \ln P_j + Q_k \ln Q_k \right).$$

When the system's configuration is amorphous, as is the case of the example represented by the configuration [2.36], the two semi-base entropies are equal to each other, each equalling *half* entropy potential *H*, as obviously evidenced - after remembering [2.37b] above - by

[2.40]
$$E_O = E_D = -\sum_{1}^{N} P_i \ln P_i = -N \left(\frac{1}{N} \ln \frac{1}{N} \right) = \ln N,$$

whence also

[2.41]
$$E^* = E_O + E_D = 2\ln N = H.$$

This equation states that the "base entropy E^*" of *any amorphous configuration* is expressed by the sum [2.41], which *is equal to the entropy potential H that is associated with the whole random interaction matrix of the amorphous "system"*.

Normally, however, the *base entropy E^** of a *generic system* is less than *H*, and the difference between *H* and E^* defines the "**base syntropy** S^*" of the system, to write

[2.42]
$$S^* = H - E^*,$$

which leads to

[2.43]
$$S + E = S^* + E^* = H, \quad \text{whence} \quad S - S^* = E^* - E \geq 0.$$

It is evident from these relations that the system's *entropy E* is *in general* less than the system's *base entropy E^**, whereas the system's **syntropy** *S* is in general greater than the system's *base syntropy S^**.

In subsequent sections of this paper, the importance of the system's *base syntropy S^** will be shown in relation to the system's *stability*.

It is now important to observe that amorphous configurations may be hidden behind interaction matrices which – at a first glance – could appear relevant to *non-amorphous* systems.

Starting by an example is still the easiest way to clarify the case.

Let's suppose that the observer of the international meeting – as addressed by the preceding example – accounts for the fact that three of the five participants in the meeting represent the governments of three South-European Countries, whereas the other two represent North-European Countries. With no other information about the meeting, *on the only basis of supposed affinities* between the meeting people, the observer may try to reduce his uncertainty through a reduction of the *would-be system* from six to three components only, grouping in his representation the South-European participants to form component "*A*"of the system, the North-European participants to form component "*B*", and the *external universe* to form component "*C*", thus re-arranging the matrix [2.36] as shown below:

x	x	x	x	x	x		
x	x	x	x	x	x	$\rightarrow$	$18x$	O_A
x	x	x	x	x	x		
x	x	x	x	x	x		
x	x	x	x	x	x	$\rightarrow$	$12x$	O_B
x	x	x	x	x	x	$\rightarrow$	$6x$	O_C
$\downarrow$	$\downarrow$	$\downarrow$		$\downarrow$			
$18x$	$12x$	$6x$	$\rightarrow$	$T = 36x$	$Tot.$		
D_A	D_B	D_C		$Tot.$			

which can be summarized by the following configuration:

$$
[2.44] \qquad
\begin{array}{c|ccc|cc}
 & A & B & C & & \\
\hline
A & 9x & 6x & 3x & 18x & O_A \\
B & 6x & 4x & 2x & 12x & O_B \\
C & 3x & 2x & 1x & 6x & O_C \\
\hline
 & 18x & 12x & 6x & 36x = T & \\
 & D_A & D_B & D_C & &
\end{array}
$$

In this example, it must be considered that no additional information allows the observer to modify his uncertainty as far as the interpersonal interactions at the meeting are concerned, so that the mean interaction between *individual* participants cannot be supposed to be different from the uniform undetermined amount *x*.

There is, however, a *new interaction distribution* between the *groups* that form the *new components* of the *potential* system *imagined* by the observer, as a consequence of the *conceptual aggregation* of both persons and flows made by the observer.

The interaction matrix in [2.44] is symmetrical, and the *probability distribution* that corresponds to this new flow distribution is given by the following series: 9/36, 6/36, 3/36, (relative to *A*); 6/36, 4/36, 2/36 (relative to *B*); and 3/36, 2/36, 1/36 (relative to *C*). The sum of these probabilities is obviously equal to 1.

By application of Shannon's formula for **entropy** one obtains:

$$
[2.45] \qquad E_{ABC} = -[(9/36)\ln(9/36) + (6/36)\ln(6/36) + (3/36)\ln(3/36) +
$$
$$
+ (6/36)\ln(6/36) + (4/36)\ln(4/36) + (2/36)\ln(2/36) +
$$
$$
+ (3/36)\ln(3/36) + (2/36)\ln(2/36) + (1/36)\ln(1/36)] =
$$
$$
= 2.02281
$$

The **entropy potential** relative to this new system of 3 components is given by

$$
[2.46] \qquad H_{ABC} = \ln(3^2) = 2\ln3 = 2.19722 ,
$$

which, as expected, is greater than entropy E_{ABC} calculated by [2.45] for the new interaction matrix of [2.44]. In this case, at variance with the previous amorphous configuration of six components,

the system's entropy does not coincide any more with the entropy potential. The difference between the two quantities, is now expressed by *syntropy*

[2.47] $$S_{ABC} = H_{ABC} - E_{ABC} = 2.19722 - 2.02288 = 0.17434 ,$$

which represents the *conceptual amount of order* that has been introduced by the observer in reshaping the six-component system into another system of three components only.

   Moreover, it is interesting as well as important to remark that the transfer flows that form the interaction matrix of configuration [2.44] are all expressed by equations of the following kind

[2.48] $$T_{jk}^* = \frac{O_j D_k}{T}; \quad (j, k = A, B, C) ;$$

i.e., in this case,

[2.49] $$T_{AA}^* = \frac{O_A D_A}{T} = \frac{18 \times 18}{36} x = 9x ; \quad T_{BB}^* = \frac{O_B D_B}{T} = \frac{12 \times 12}{36} x = 4x ;$$

$$T_{CC}^* = \frac{O_C D_C}{T} = \frac{6 \times 6}{36} x = 1$$

[2.50] $$T_{AB}^* = \frac{O_A D_B}{T} = \frac{18 \times 12}{36} x = T_{BA}^* = 6x$$

$$T_{AC}^* = \frac{O_A D_C}{T} = \frac{18 \times 6}{36} x = T_{CA}^* = 3x ;$$

[2.51] $$T_{BC}^* = \frac{O_B D_C}{T} x = \frac{12 \times 6}{36} x = T_{CB}^* = 2x$$

   Let's now calculate the *base entropy* of the new configuration [2.44] suggested by the observer.
   Because of the identity of the *origin-semi base* with the *destination semi-base*, the *base entropy* of that configuration is given by

[2.52] $$E^* = -2 \times \left(\frac{O_A}{T} \ln \frac{O_A}{T} + \frac{O_B}{T} \ln \frac{O_B}{T} + \frac{O_C}{T} \ln \frac{O_C}{T} \right) =$$

$$-2(0.5\ln 0.5 + 0.33333\ln 0.33333 + 0.16666\ln 0.16666) =$$
$$= 2.02281 ;$$

which coincides with entropy E_{ABC} calculated in [2.45] above.

2.8 *Some conclusions*

(*1*) In the three-component configuration [2.44] the entropy E_{ABC} associated with the interaction distribution $\{T_{jk}^*\}$ coincides with the relevant base entropy E^*, as it is also of the original *amorphous* configuration [2.36]. As proved in the next chapter, this fact is true *in general*, when – given a configuration whatever – all the elements of the *random* interaction matrix $\{T_{jk}^*\}$ are obtained through equations of the kind introduced by [2.48] above.

The principal meaning of the preceding example is that no artful manipulation of the symbols representing an *amorphous* configuration can change the nature of this: A *full uncertainty* about any observed set of "potential components" of a *would-be system* can never represent a *viable* system.

(*2*) *The same base configuration* can be associated with *all those systems which have an identical base.*

One example may be sufficient to show what this means. Consider the following two configurations relative to two different systems both sized $N=3$.

[2.53]

1.	*Configuration upon survey*			
	A	*B*	*C*	O_i
A	13.5	8.2	3.3	25.0
B	7.4	9.0	6.6	23.0
C	2.1	7.3	12.1	21.5
D_k	23.0	24.5	22.0	69.5

2.	*Configuration as per equations* [2.48]			
	A*	*B**	*C**	O_i
A*	8.273	8.813	7.914	25.0
B*	7.612	8.108	7.280	23.0
C*	7.115	7.579	6.806	21.5
D_k	23.0	24.5	22.0	69.5

Both configurations share the same *base configuration* and – therefore – also the same *base entropy* and *base syntropy*, given (remember [2.39]) by

[2.54]
$$E^* = -\left(\sum P_j \ln P_j + \sum Q_k \ln Q_k \right) \text{ and } S^* = H - E^*,$$

respectively, in which $\{P_j\}$ and $\{Q_k\}$ are defined by [2.13] and [2.14], respectively.

The first of these two configurations represents a system whose interactions have been detected on the field, directly; whereas the *active matrix* of the second configuration has been calculated by means of the "random flow equations" introduced with [2.48], using the *base* of the first configuration, as evidenced by the bold font.

The second configuration, however, represents only a "reshaping" of an *amorphous configuration* of *random flows*. The entropy of the *interaction matrix* of this *random configuration* is in fact $E_3^* = 2.194334$, which coincides with the *base entropy* E_3^* shared by both the configurations [2.53]. The corresponding *base syntropy* is therefore given by

$$S_3^* = H_3 - E_3^* = 2\ln3 - 2.194334 = 0.00289.$$

The entropy associated with the *active matrix* of the configuration *detected on the field* is instead $E_3 = 2.088692$.

The corresponding syntropy is $S_3 = 0.10853$.

As it is clear, an infinite number of possible systems sized $N=3$ can be associated with the same *random configuration*, which is therefore dubbed "*the base configuration*" of any other possible configuration with the same base.

Once established *the base*, the entropy *E* of all possible systems sharing the same base is necessarily *less* than the entropy E^* of the associated *base configuration*, which entails that the syntropy *S* (degree of order) of all those systems is obviously *greater* than the syntropy S^* pertaining to the relevant *base configuration*, as stated by the relations [2.43]; whereas the *entropy potential H* = 2ln*N* keeps constant with *N* for all possible configurations sized *N*.

(3) **As another important conclusion**, there is to learn that not only can not the entropy of any system exceed the entropy of its *base configuration*, but also, perhaps surprisingly, that:

(a) whatever the configuration of any system among all those that share a *common base*, the system's achievable *maximum level* of organization (*syntropy*) is *numerically equivalent* to the *entropy level* E^* of the relevant *base configuration*;

(b) whatever the configuration of any system among all those that share a *common base*, the system's entropy can never collapse below the *level numerically equivalent* to the *syntropy* S^* *level* of the relevant *base configuration*.

Therefore, any *base configuration* fixes the coincident limits of maximum entropy/syntropy and corresponding minimums for the infinite variety of systems whose configurations share the same *base*.

It is also important to remark that the common *base configuration* shared by the various systems shall *basically* be understood, along with the relevant system configurations, in terms of *probability distributions*; which - in particular cases - allows the various systems to differ from each other in terms of *volumes* of the transfer interactions. For - if one multiplies *all* the system's flows by the same (any) factor *a* - no change occurs in the probability distribution of its active matrix as well as in its entropy and in its syntropy.

The multiplication of all the *elements* of the system by the same factor *a* may be referred to as the "*a*-anamorphism" of the system.

(4) **As a major conclusion**, one can state – on the basis of the concepts formulated so far – that any set of interacting "components" (as identified by the observer's attention) forms a "viable system" *only if the measured intensities* of some (or all) of the observed interaction flows reveal significant differences with the values calculated for the same interactions by use of the *random flow equations* [2.48].

2.9 *A "family" of evolution ellipses*

From the preceding conclusions it is now possible to draft a graph that may represent a "family" of evolution paths for any system sized *N*.

Figure 2.1 in preceding Paragraph 2.5 has provided a first example of evolution ellipse. It concerns a geometrical representation of all the possible states of an evolving system which keeps its base configuration constant. Actually, the base configuration determines the *eccentricity* of the evolution ellipse and – therefore - of the ellipse's shape. The *eccentricity* depends of the position (distance) of the *fouces* from the centre of the ellipse: The greater the base entropy, the greater the eccentricity, which is numerically quantified by the following difference (see also Figure 2.2 below):

[2.55] $$\eta = \frac{\ln N - S^*}{\ln N} = 1 - \frac{S^*}{\ln N} \ .$$

For example, with reference to the configurations in [2.53], the eccentricity of the relevant ellipse is very high, as it is expressed by

$$\eta = 1 - 0.00289/1.09861 = 1 - 0.00263 = 0.99737 \ .$$

Such a high eccentricity entails a very "flat" ellipse. The flatter the ellipse the greater the level of organization that the system *may* achieve on the given *base configuration*.

FIGURE 2.2
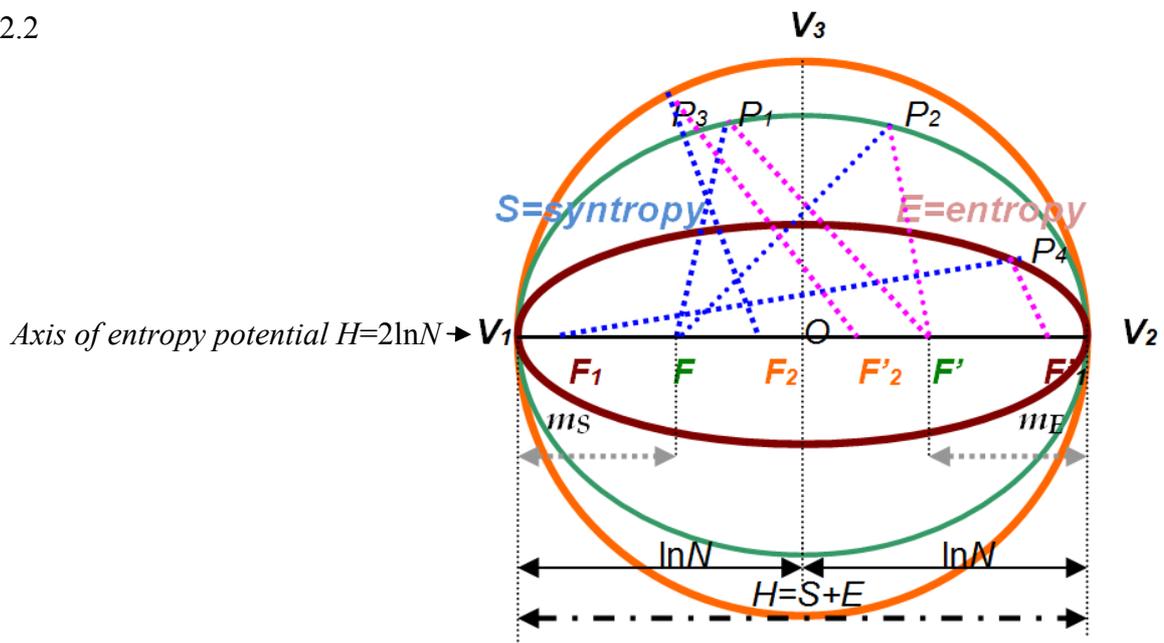
The three ellipses of Figure 2.2 represent three different evolution paths for systems with an identical entropy potential $H = 2\ln N$. This potential is representd by the ellipse's major axis V_1V_2 .

Points P_1, P_2, P_3, P_4 may also represent four different possible states of an evolving system. The coupled distances of these points from the two focuses of the respective ellipses represent the relevant amounts of entropy (blue-colour dotted lines) and syntropy (violet-colour dotted lines). As per the definition of "ellipse", the sum of the coupled distances is always equal to the entropy potential H.

The segments generically indicated by symbols m_S and m_E represent the *minimum syntropy* (coincident with the relevant *base syntropy* S^*) and the *minimum entropy*, respectively, relative to the evolving systems. (As an example, in the figure $m_S = FV_1$ and $m_E = F'V_2$ represent the minimum syntropy and entropy, respectively, *relative to the green-colour evolution path*).

Distances OF, OF_1, OF_2 with the "*F*"s in different colour font are the *focal distances* relative to the ellipses of the respective colour.

It is of a major importance to point out that the system's evolution does not necessarily keep the system on the same "elliptical" path, since the evolution is normally characterized by changes in the *base configuration of the system*. However, if the changes that intervene in the base configuration

do not involve changes in the *value of the base entropy*, then the system proceeds on the same elliptical evolution path.

Actually, it should be clear that when the system's configuration changes *with no change in the value of its base entropy*, the same *base entropy* may be associated with an infinite number of different *base configurations* that pertain to the same system; albeit the constancy in the value of the base entropy shall be considered as rather an exceptional aspect of the system's evolution process.

As a detail, which may also help draft the geometrical figure of an evolution path, consider that the semi-minor-axis *b* of any evolution ellipse can be calculated as

[2.56]
$$b = \sqrt{\ln^2 N - (\ln N - S^*)^2} = \sqrt{S^*(H - S^*)} = \sqrt{S^* E^*} \ ,$$

in which $\ln N - S^*$ is the distance of each focus from the ellipse's centre *O*, while bearing in mind that S^* and E^* in this formula are the *base syntropy* and the *base entropy*, respectively, relative to the system considered.

3. Interaction flow equations – The system's structure

3.1 *The random flow equation*

In the preceding chapter, the equations [2.48] in Paragraph 2.7 have heuristically introduced the formula for calculating the *random flows* $\{T_{jk}^*\}$ inherent in any given *amorphous configuration*. The associated *random probability matrix* $\{P_{jk}^*\}$ consists – by definition, remembering [2.12] – of the elements expressed by

[3.1]
$$P_{jk}^* = \frac{T_{jk}^*}{T} = \frac{O_j D_k}{T^2} ; \qquad\qquad (\forall j, k).$$

Actually, it is a definition of probabilities obtained as "compound probabilities". A *compound probability* is the probability of coincident occurrence of two events having independent probabilities of occurrence. Definitions [2.13] and [2.14] have introduced P_j as the probability that any flow in the system is generated by source *j*, and Q_k as the probability that any flow is bound for destination *k*. In absence of any other information concerning the flows, the theory of probability states that the probability for any flow to be (in the given time unit) both originated in *j* and bound for *k* shall be calculated as the "compound probability" expressed by the product of probability P_j and probability Q_k, *i.e.*, by

[3.1a]
$$P_{jk}^* = P_j Q_k = \frac{O_j}{T} \times \frac{D_k}{T} = \frac{O_j D_k}{T^2} = \frac{T_{jk}^*}{T} ; \qquad (\forall j, k);$$

whence also the equations

[3.1b]
$$T_{jk}^* = P_{jk}^* T = \frac{O_j D_k}{T} \ ; \qquad\qquad (\forall j, k),$$

which coincide with the heuristically obtained equations [2.48].

These equations regard all possible *amorphous systems*, as already observed in the previous chapter.

The equations for the determination of a random probability distribution $\{P_{jk}^*\}$ can also be obtained by considering that any random configuration may be *variously constrained* by the shape of its base, *i.e.,* by its matrices $\{P_j\}$ and $\{Q_k\}$, which depend on how the random flows of the initial amorphous configuration have been aggregated by the analyst, whatever the purpose. Recalling the definitions [2.9], [2.10], [2.11], the probabilities to assess are subject to the following set of constraints:

[3.2]
$$\psi_1 = \sum_{j,k} P_{jk}^* - 1 = 0 \,,$$

[3.3]
$$\psi_2^{\,j} = \sum_{k} P_{jk}^* - P_j = 0 \,; \qquad\qquad (\forall\, j)$$

[3.4]
$$\psi_3^{\,k} = \sum_{j} P_{jk}^* - Q_k = 0 \,; \qquad\qquad (\forall\, k),$$

in which $\{P_j\}$ and $\{Q_k\}$ are supposed to be known. However, these equations are not sufficient for an algebraic determination of $\{P_{jk}^*\}$, because the number of the *2N independent constraint equations* [3.2], [3.3], [3.4] *is always less than the number of the N^2 unknowns* [21]. In such cases, as to algebra, the possible solutions to the problem are of no use because they are infinite in number, thus leaving the problem *undetermined*.

Apart from the constraints [3.2] to [3.4], no other condition *constrains* the interaction probabilities of the amorphous configuration, so that the remaining *uncertainty* (as also reflected by the relevant algebraic indetermination) may be viewed as the remaining degree of disorder that characterizes the interactions' behavior. Then, the problem of determining the most logical values for the random flow probabilities in a "conditioned" amorphous configuration is of the same nature as that for the determination of the random flow probabilities $\{\bar{P}_{jk}\} = \{1/N^2\}$ obtained by equation [2.32a]. Therefore, the problem is to determine the $\{P_{jk}^*\}$ that allow the amorphous system to achieve its maximum level of entropy (disorder) E^* under the constraints [3.2], [3.3] and [3.4].

Using the Lagrangian multipliers method again, the relative *Lagrangian function* L^* to write is:

[3.5]
$$L^* = \alpha\psi_1 + \sum \beta_j\, \psi_2^{\,j} + \sum \gamma_k\, \psi_3^{\,k} + E^* \,,$$

in which the *2N+1* coefficients α, β_j and γ_k are the *Lagrangian multipliers* to be determined in function of the values found for $\{P_{jk}^*\}$.

The coordinates $\{P_{jk}^*\}$ that identify the point of maximum value of the constrained entropy $E^* = -\sum_{j,k} P_{jk}^* \ln P_{jk}^*$ must satisfy the condition expressed by the following N^2 equations:

[3.6]
$$\frac{\partial L^*}{\partial P_{jk}^*} = 0 \,, \qquad\qquad (\forall\, j,\, k);$$

---

[21] With the exception of the non-significant case of a system consisting of two components only.

which means

[3.7] $\qquad \dfrac{\partial L^*}{\partial P^*_{jk}} = \alpha + \beta_j + \gamma_k - 1 - \ln P^*_{jk} = 0$, $(\forall j, k)$;

from which

[3.8] $\qquad\qquad\qquad P^*_{jk} = e^{\alpha-1} e^{\beta_j} e^{\gamma_k}$; $(\forall j, k)$.

By substitution of this expression for P^*_{jk} in the constraint equation [3.2] one obtains

[3.9] $\qquad\qquad e^{\alpha-1} = \dfrac{1}{\sum e^{\beta_j} \sum e^{\gamma_k}}$,

which can in turn be used in [3.8], leading to

[3.10] $\qquad P_j = \dfrac{e^{\beta_j} \sum e^{\gamma_k}}{\sum e^{\beta_j} \sum e^{\gamma_k}} = \dfrac{e^{\beta_j}}{\sum e^{\beta_j}}$, $(\forall j)$,

whence also

[3.11] $\qquad\qquad e^{\beta_j} = P_j \sum e^{\beta_j} = AP_j$, $(\forall j)$,

$A = \sum e^{\beta_j}$ being a constant, whatever the system's *component* regarded.

In quite an analogous way, with reference to the $\{Q_k\}$, one can obtain

[3.12] $\qquad\qquad e^{\gamma_k} = Q_k \sum e^{\gamma_k} = BQ_k$, $(\forall k)$,

$B = \sum e^{\gamma_k}$ being also a constant, whatever the *component* regarded. Then, by substitution of the

exponentials of the equation [3.8] with [3.9], [3.11] and [3.12], respectively, the searched solution to the problem is given by

[3.13] $\qquad\qquad P^*_{jk} = \dfrac{AP_j BQ_k}{AB} = P_j Q_k = \dfrac{O_j D_k}{T^2}$; $(\forall j, k)$.

The method, which has been followed to arrive at this solution, leads to the confirmation of the previous equations [3.1] found for the *probability* of *random interaction flows* as well as for the *probable* flow equations [3.1b].

It is worth remarking that the same result as that shown by equations [3.13] is obviously obtained if a uniform mean *efficacy* $u^* = U/T$ is associated with each unit interaction flow of the system, in order to write the constraint equation [3.2] also in the following way:

[3.2a] $\qquad\qquad \psi_1' = \sum_{j,k} u^* P^*_{jk} - u^* = 0$,

($U = u^* T$ is the supposed overall "efficacy" of the system's activity).

Actually, there is to admit that a *conventional* "unit" mean uniform efficacy $u^* = 1$ has virtually been allowed for by constraint equation [3.2]. Then, what if the uniform efficacy is $u^* = 0$? The answer is that no substantial change modifies the result given by equations [3.13]: The first

constraint equation ψ_1 would disappear as a constraint, but it would be re-introduced as a linear combination of the remaining two sets of constraint equations ψ_2 and ψ_3, in order to let the left hand side of equation [3.9] become e^{-1} instead of $e^{\alpha-1}$, while all the subsequent steps, up to the conclusion [3.13], would remain unchanged.

The preceding remark is worth the confirmation that the random flow distribution is not intrinsic to the system observed, but only the expression of the analyst's maximum uncertainty (or lack of information) about the *causes* of the system's activity.

3.2 *The equation of the "intentional" interaction flow*

The situation described by the analysis carried out in the preceding paragraph changes remarkably if – in general – the *mean* unit *efficacy* of the interactions varies with each of the interaction flows.

As discussed in Paragraph 2.3 of the preceding chapter, the analyst, whose purpose is to describe a *complex adaptive system*, does implicitly assumes that the system's components attach the expectation of some "programmed" effects to the interactions they generate. Indeed, the analysis concerns the representation of systems consisting of *intentional interaction flows*.

Nevertheless, nothing can assure the analyst that the "efficacy" of every interaction flow of the system attains the expected level. On the contrary, daily common experience as well as scientific observations concerning both human interaction systems and any other kind of biologic interaction system, show that only part of the interaction activities meets the activators' expectations completely. A large number of the "results" obtained through the interaction activity are unpredicted, to the extent to which they reveal incomplete or null or even detrimental to the system's involved components. Perhaps, it is just such an *endemic* uncertainty, which affects the interaction efficacy, the factor that compels complex systems to become *adaptive* for survival.

The awareness of this condition justifies the adoption of the Lagrangian multiplier method also for the determination of the *intentional flow equation*, accounting for a "**generic mean efficacy** u_{jk}" supposedly asso-ciated with and proper to each *unit* of each intentional flow T_{jk} .

Therefore, with a view to assessing *in general* the probability of occurrence $P_{jk} = T_{jk}/T$ for each interaction flow, *the mathematical problem* to solve can be formulated as follows: Find the $\{P_{jk}\}$ that *maximize* the *uncertainty function* $E = E(P_{jk})$, under the constraints imposed by all the information that is - *in general* [22] - available to the analyst about the interaction system. Translated into formulas, it is:

[3.14]
$$E = -\sum_{j,k} P_{jk} \ln P_{jk} = maximum$$

constrained by

[3.15]
$$\varphi_1 = \sum_{j,k} u_{jk} P_{jk} - u = 0 \, ,$$

[3.16]
$$\varphi_2{}^j = \sum_{k} P_{jk} - P_j = 0 \, ; \qquad (\forall j)$$

[3.17]
$$\varphi_3{}^k = \sum_{j} P_{jk} - Q_k = 0 \, ; \qquad (\forall k),$$

---

[22] In particular applications of the theory, the information available to the analyst might subject groups of variables to different or additional constraints.

in which $\{P_j\}$ and $\{Q_k\}$ are supposed to be known, whereas both the term $u = U/T$ of φ_1 and the matrix of constant coefficients $\{u_{jk}\}$ are actually undetermined and are here used as logical instruments only.

(Once in a while it is worth reminding the reader that the analysis does not regard the *real reasons* for the interactions observed, but regards only a way of reasoning upon subjective observations).

The Lagrangian L relative to the uncertainty E to maximize is now

[3.18] $$L = \lambda\varphi_1 + \sum \theta_j\ \varphi_2^{\ j} + \sum \vartheta_k\ \varphi_3^{\ k} + E\ ,$$

where λ, θ_j, ϑ_k are the $N+1$ *Lagrangian multipliers* associated with the respective $N+1$ constraint equations φ_1, $\varphi_2^{\ j}$, $\varphi_3^{\ k}$.

Again, the necessary condition required for the solution to the problem is expressed by the following set of N^2 equations:

[3.19] $$\frac{\partial L}{\partial P_{jk}} = 0\ , \qquad\qquad (\forall\ j,\ k);$$

which means

[3.20] $$\frac{\partial L}{\partial P_{jk}} = \lambda u_{jk} + \theta_j + \vartheta_k - 1 - \ln P_{jk} = 0\ , \qquad\qquad (\forall\ j,\ k)\ ;$$

from which, after writing $\theta_j - 1 = \alpha_j$, one obtains

[3.21] $$P_{jk} = e^{\alpha_j} e^{\vartheta_k} e^{\lambda u_{jk}}\ ; \qquad\qquad (\forall\ j,\ k).$$

The exponentials of the Lagrangian multipliers are now obtained by replacing the $\{P_{jk}\}$ in the constraint equations with the corresponding values expressed by [3.21]. In particular, with the substitution in constraint equation [3.16], one obtains

[3.22] $$P_j = e^{\alpha_j} \sum_k e^{\vartheta_k} e^{\lambda u_{jk}}\ , \qquad\qquad (\forall\ j);$$

from which

[3.23] $$e^{\alpha_j} = \frac{P_j}{\sum_k e^{\vartheta_k} e^{\lambda u_{jk}}} = A_j P_j\ ; \qquad\qquad (\forall\ j).$$

Then, by substitution with [3.21] in [3.17]:

[3.24] $$Q_k = e^{\vartheta_k} \sum_j e^{\alpha_j} e^{\lambda u_{jk}}\ , \qquad\qquad (\forall\ k);$$

from which

[3.25] $$e^{\vartheta_k} = \frac{Q_k}{\sum_j e^{\theta_j} e^{\lambda u_{jk}}} = B_k Q_k \qquad\qquad (\forall\ k).$$

Coefficients $\{A_j\}$ and $\{B_k\}$, whose definition is here obvious, have been introduced for writing simplification only. The relations [3.23] and [3.25] can now be used to replace the respective exponentials in [3.21], to write:

[3.21a] $P_{jk} = A_j B_k P_{jk}^* e^{\lambda u_{jk}}$,   ($\forall j, k$),

after remembering – as found in [3.13] – that $P_{jk}^* = P_j Q_k$, ($\forall j, k$)**.**

From [3.21a], after multiplication by the total amount T of detected flows, one derives the equation of the most probable distribution of *intentional interaction flows* in the system observed, to write

[3.21b] $T_{jk} = A_j B_k \dfrac{O_j D_k}{T} e^{\lambda u_{jk}}$,   ($\forall j, k$),

The determination of the 2N coefficients $\{A_j\}$ and $\{B_k\}$ is a problem to discuss.

Suppose that the $\{\mu_{jk}\}$ are known and the actual $\{P_{jk}\}$ have been determined after the *baseline survey* on the field, which provides the data for the calibration of coefficients $\{A_j\}$, $\{B_k\}$ and Lagrangian multiplier λ. After passing from equations [3.21a] to the relevant logarithm, one obtains

[3.26] $\ln A_j + \ln B_k + \lambda u_{jk} = \ln \dfrac{P_{jk}}{P_{jk}^*}$;   ($\forall j, k$).

It is a system of $N.^2$ inhomogeneous linear equations in the 2N+1 unknowns $\ln A_j$, $\ln B_k$ and λ, whereas $\{P_{jk}\}$ and $\{ P_{jk}^* \}$ - which determine the equations' known terms – result from the survey data.

The number of the equations is far exceeding the number of the unknowns, and algebra states that the solution to the problem is impossible if all the equations are independent of each other. Which is not true as to our case: It can be proved that all the matrix determinants of (2N+1)×(2N+1) order are null, because of intervening linear combinations affecting columns of the matrix. However, it is also impossible to state that *univocal* solution values for the unknowns can be determined, whatever the matrix of the equation system's coefficient. Actually, in the coefficient matrix of equations [3.26] it is always possible to find non-null *minors* of 2N× 2N order, so that solutions to the problem can be obtained by assigning arbitrary values to any of the unknowns of the independent equations. The number of possible solutions is infinite.

An empirical – though correct method for establishing workable values for the unknowns, successfully tested in most similar cases, could be that of a reiterative procedure for each of the equations [3.21a], after fixing an arbitrary value for one of the relevant unknowns.

For sure, the use of the intentional flow equations in the form provided by [3.21a] or [3.21b] is rather inconvenient for the analyst.

One logical remedy is considering that it is always possible to find suitable arbitrary values for *either* $\{A_j\}$ *or* $\{B_k\}$ in order to obtain

[3.26] $\ln A_j + \ln B_k = \ln(A_j B_k) = \ln h = constant,$ ($\forall j, k$).

Constant parameter h will be referred to as "the system's **state factor**".

Then, by substitution in [3.21a] and in [3.21b], the *intentional flow* **probability** *equations* and the *intentional* **probable flow** *equations* become, respectively,

[3.21c]
$$P_{jk} = h P_{jk}^* e^{\lambda u_{jk}} \,, \qquad\qquad (\forall\, j,\, k),$$

and

[3.21d]
$$T_{jk} = h \frac{O_j D_k}{T} e^{\lambda u_{jk}} = h T P_{jk}^* e^{\lambda u_{jk}} \,, \qquad\qquad (\forall\, j,\, k).$$

From either sets of equations one obtains:

[3.27]
$$h = \frac{1}{\sum\limits_{j,k} P_{jk}^* e^{\lambda u_{jk}}} \,,$$

which, replacing *h* in [3.21.c], leads to the Bayes' equations for *conditional probabilities*, as expressed by

[3.21e
$$P_{jk} = \frac{P_{jk}^* e^{\lambda u_{jk}}}{\sum\limits_{j,k} P_{jk}^* e^{\lambda u_{jk}}} \,; \qquad\qquad (\forall\, j,\, k).$$

Actually, as I could show elsewhere,[23] equations [3.21e] can be obtained also through the Bayesian inference that in the theory of probability concerns the *conditional probabilities* of a distribution.

The interaction flow equations formulated so far regard *standing* (*unstable*) *states* of the system, since the assessment of the interaction flows and relevant probabilities is based on the findings of a survey: This is carried out upon the implicit assumption that the system is conventionally supposed to be in a *state of temporary equilibrium*, in the obvious awareness that the system evolves. In any case, the same equations shall be considered as the *baseline equations* that describe the *original standing state* of the system's evolution.

3.3 *The intents associated with flows – Relation coefficients*

The foregoing paragraph has omitted comments on the determination of λ, the third Lagrangian multiplier of the equations [3.20] and [3.26]. The value of this parameter depends on the measurement units used for quantifying the system's interaction flows: Its basic role is providing the relevant exponentials with a counterbalancing dimensional factor that makes $\{\lambda u_{jk}\}$ a matrix-set of quantities with no physical dimension. For the purposes of the theory, the isolated determination of λ is of quite a secondary importance, whereas these exponents, *i.e.*, the matrix $\{\lambda u_{jk}\}$ – as announced with the definition [2.17] introduced in Chapter 2 – are in themselves more important as

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[23] M. Ludovico, *L'evoluzione sintropica dei sistemi urbani*, Bulzoni, Rome 1988 -1991, pp. 89 to 98.

matrix of the "**intents**" that form the "**structure**" of the system. The respective exponentials, given by

[3.28] $$\varepsilon_{jk} = e^{\lambda u_{jk}} = e^{\mu_{jk}} \ , \qquad (\forall j, k),$$

constitute the matrix $\{ e^{\mu_{jk}} \} = \{ \varepsilon_{jk} \}$ of the system's "**relation coefficients**"**,** as obviously determined – considering either equations [3.21c] or [3.21d] – by

[3.29] $$\varepsilon_{jk} = e^{\mu_{jk}} = \frac{P_{jk}}{h P_{jk}^*} = \frac{T_{jk}}{h T_{jk}^*} = \frac{R_{jk}}{h} \ , \qquad (\forall j, k),$$

in which the ratios $R_{jk.} = P_{jk} / P_{jk.}^* = T_{jk} / T_{jk}^*$ will be referred to as "**flow quotients**", for they represent the *deviation* of the *actual* interaction flows with respect to the respective theoretical *random flows* of the system's *base configuration*.

As already introduced in Chapter 2, both the *intent matrix* $\{\mu_{jk}\}$ and the *relation matrix* $\{\varepsilon_{jk}\}$ may indifferently be referred to as "**the system's structure**".

From the foregoing definitions, the *intents* of the system's interaction units are expressed in function of the system's flows and *state factor h*: From [3.29] one obtains

[3.30] $$\mu_{jk} = \ln \left(\frac{R_{jk}}{h} \right) = \ln \left(\frac{T_{jk}}{T_{jk}^*} \right) - \ln h \ ; \qquad (\forall j, k) \ .$$

These equations give the opportunity to identify the meaning of *state factor h*. There is in general to consider that the *flow ratios* R_{jk} may have any value, including 1, when one or more flow probabilities coincide with the respective probabilities of the system's base configuration (*e.g.* $P_{ab} = P_{ab}^*$ or/and $P_{cd} = P_{cd}^*$, etc.). Which is obviously possible, considering that not all of the system's mean interaction "intents" differ necessarily from the *mean uniform intent* that can be associated with every interaction unit of the respective base configuration.

Therefore, when $R_{jk.} = 1$ the equations [3.30] tell us that the value for *h* is also expressed by

[3.31] $$\mu^* = \ln 1 - \ln h = - \ln h \ ,$$

in which μ^* is *the mean uniform intent* associated with every interaction unit of the system's base configuration. As a consequence, the equation [3.30] can also be written

[3.30a] $$\mu_{jk} = \ln \left(\frac{R_{jk}}{h} \right) = \ln \left(\frac{T_{jk}}{T_{jk}^*} \right) + \mu^* \ ; \qquad (\forall j, k)$$

and the equations [3.21c] and [3.21d] can also be written as follows:

[3.21e] $$P_{jk} = P_{jk}^* e^{\lambda(u_{jk} - u^*)} \ , \qquad (\forall j, k),$$
and
[3.21f] $$T_{jk} = T P_{jk}^* e^{\lambda(u_{jk} - u^*)} \ , \qquad (\forall j, k) \ .$$

In fact, from equation [3.31], it is immediately evident that

[3.31a] $$h = e^{-\mu^*} = e^{-\lambda u^*},$$

with u^* representing again the mean uniform *efficacy* of the *base configuration*'s interaction flows. In this connection, it is worth observing that the general equations [3.1] and [3.13] for the *random flow probabilities* can also be written as

[3.13a] $$P_{jk}^* = h \frac{O_j D_k}{T^2} e^{\mu^*} \left(= \frac{O_j D_k e^{\mu^*}}{T^2 e^{\mu^*}} = \frac{O_j D_k}{T^2} \right), \qquad (\forall\, j,\, k).$$

Note also that equation [3.31a] expresses the only value for h that confirms Bayes' equations [3.21e] for the *conditional probabilities* of the distribution.

3.4 *The structure of the system*

The discussion about the meaning of the *state factor h* is a way for introducing the concept, proper to the theory, of *the system's structure.*

Equation [3.31] draws attention to the fact, previously noted [24], that the idea of a *non-null intent* is implicitly associated by the analyst also with the random flows of amorphous configurations, in particular with the flows of the system's base configuration. Indeed, the identification of *active components* that *differ from one another* depends on the cultural baggage of the observer. The aptitude for *recognizing* any system is a *shaping activity* of human intelligence, which can seize the existence of relationships between the identified *components*. Such relationships entail *interaction flows* whose intent is *non null in principle,* but it is instead unknown because of a total lack of information about the supposedly existing interactions. *"Uniform intent",* as indicated by the equation [3.31] means only that the observer cannot at once distinguish the interactions' intents from one another.

In turn, the *shape* of the system's body, as formed by the identified and selected *components*, bears in itself the amount of information inherent in the observer's choice. The *shape* of any system is primarily given by how the interactions concentrate in the relevant sources and destinations, as shown by the figures that form the *base* of the system. Such a shape *constrains* the distribution of the *random interaction flows* whose intensity – for probability reasons – is in all cases directly proportional to the *size/importance* of both the sources and the destinations of the interactions.

The degree of order, the *base syntropy* of any *base configuration* coincides with the syntropy associated with the distribution of the interactions among the sources and the destinations of the "random" system. It is a coincidence that induces one to recognize that the *uniform mean intent* associated with every unit of the "random flows" is just the "necessary" *basic unit intent* that *the observer implicitly assigns* to the interactions of the system he shapes.

In other terms, the uniform mean intent expresses the *"natural" minimum need for organization* that can justify the *recognition* of the system made by the observer; who is induced to guess (as it will later be confirmed) not only that "his" system entails a direct proportionality between "basic

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[24] Remember the comment on constraint equation [3.2a], Paragraph 3.1, p. 48, as well as the previous comment on inequalities [2.35] in Chapter 2, p. 33.

intent" and *base syntropy*, but also that the coefficient of proportionality is equal to 1, in order to write

[3.32] $$\mu^* = -\ln h = S^* \ ,$$

considering that the dimension-less coefficient – which connects two dimension-less quantities - does not depend on the measurement system adopted for the intents. Therefore, also:

[3.33] $$h = \frac{1}{e^{\mu^*}} = \frac{1}{\sum\limits_{j,k} P_{jk}^* e^{\mu_{jk}}} = \frac{1}{e^{S^*}} \quad ,$$

which helps clarify the meaning of "state factor" *h***:** This parameter is equal to "1" only in the "boundary" case of $\{\overline{P}_{jk}^*\} = \{1/N^2\}$, in which case $E^* = E = \ln N^2$ and $S^* = S = \mu^* = 0$; whereas, in all other cases, it is

[3.34] $$h = e^{-S^*} = e^{-(\ln N^2 - E^*)} = \frac{e^{E^*}}{N^2} \to 0 \quad \text{as} \quad N \to \infty \ .$$

Therefore, values for *h* are possible *only* according to the following range of values

[3.35] $$0 < h < 1 \ ,$$

because the numerical value for e^{E^*} can neither exceed nor attain $N.^2$, below which its ratio to $N.^2$ is always less than 1 and tends to zero as N tends to infinite.

3.4.1 *Structure potentials*

Remembering the definition [3.28] and the equations [3.13] and [3.21c], one can write

[3.36] $$P_{jk} = hP_{jk}^* e^{\lambda u_{jk}} = h\frac{O_j D_k}{T^2} e^{\mu_{jk}} = hP_j Q_k \varepsilon_{jk} \ , \qquad (\forall j, k)$$

By separate sums of these equations, first on *k* and after on *j,* one gets

[3.37] $$\sum_k Q_k \varepsilon_{jk} = \frac{1}{h} \quad \text{and} \quad \sum_j P_j \varepsilon_{jk} = \frac{1}{h} \ , \quad \text{respectively,} \quad (\forall j, k).$$

From which, clearly, also

[3.38] $$\sum_j hP_j \varepsilon_{jk} = \sum_k hQ_k \varepsilon_{jk} = 1 \ , \qquad (\forall j, k) \ .$$

After setting

[3.39] $$Y_j = hP_j, \ (\forall j), \quad \text{and} \quad X_k = hQ_k \ , \qquad (\forall k),$$

the two systems [3.37] of N equations each become

[3.37a] $$\sum_j Y_j \varepsilon_{jk} = 1 \ , \qquad\qquad (\forall k),$$

[3.37b] $$\sum_k X_k \varepsilon_{jk} = 1 \ , \qquad\qquad (\forall j) \ .$$

Name Y_j and X_k "**structure potentials Y**" and "**structure potentials X**", respectively, and consider that **these quantities are *invariant with the system's structure* $\{\varepsilon_{jk}\}$**: The $2N$ *structure potentials X and Y* do not change with the system's configuration until the system's structure $\{\varepsilon_{jk}\}$ remains unchanged. Given the structure $\{\varepsilon_{jk}\}$, the unknown *potentials* are determined by solution of the equation systems [3.37a] and [3.37b], in which the coefficient matrix of the former system is the *transposed* of the latter.

Accounting for the definitions [3.38], it is possible to write the following two equations:

[3.40] $$h = \sum Y_j = \sum X_k \ ,$$

which regards systems in standing states *only*.

From [3.39] consider also the obvious definitions

[3.41] $$P_j = \frac{Y_j}{h} , \ (\forall j) , \quad \text{and} \quad Q_k = \frac{X_k}{h} \ , \ (\forall k) \ .$$

3.4.2 *Canonical and semi-canonical equations*

Given the system's structure, the *structure potentials* allow a different way to write both the *flow probability* equations and the *probable flow* equations.

From any of the flow equations or flow probability equations, *e.g.,* from [3.36], the use of the *structure potentials* leads to:

"**Canonical flow *probability* equations**":

[3.42] $$P_{jk} = \frac{Y_j X_k}{h} \varepsilon_{jk} \ , \qquad\qquad (\forall j, k) \ .$$

"**Canonical probable *flow* equations**":

[3.43] $$T_{jk} = \frac{T Y_j X_k}{h} \varepsilon_{jk} \ , \qquad\qquad (\forall j, k) \ .$$

Semi-canonical "**Flow *generation probability* equations**":

[3.44] $$P_{jk} = P_j X_k \varepsilon_{jk} \ , \qquad\qquad (\forall j, k) \ .$$

Semi-canonical **"Flow *destination probability* equations**":

[3.45] $$P_{jk} = Y_j Q_k \varepsilon_{jk} \ ,$$ ($\forall j, k$) .

Semi-canonical **"Flow *generation* equations**":

[3.46] $$T_{jk} = O_j X_k \varepsilon_{jk} \ ,$$ ($\forall j, k$) .

 Semi-canonical **"Flow *destination* equations**":

[3.47] $$T_{jk} = Y_j D_k \varepsilon_{jk} \ ,$$ ($\forall j, k$) .

"Canonical random flow *probability* equations":

[3.48] $$P_{jk}^* = \frac{Y_j X_k}{h^2} \ ,$$ ($\forall j, k$) .

Semi-canonical **"*random flow* equations":**

[3.49] $$T_{jk}^* = \frac{T Y_j X_k}{h^2} \ ,$$ ($\forall j, k$) .

The foregoing definitions and equations have a major role in the description of evolution processes of the system.

3.4.3 *The system's structure and stability – A few comments*

At theoretical level, the *relation coefficients* deserve the label of "**the system's structure**", although they are unknown in starting practical applications of the theory. The two preceding paragraphs have shown that the *flow intents*, in the form of *relation coefficients,* are in themselves the "cause" of the system's interactions. Configurations and viability of systems are substantially depending on the set of the relevant relation coefficients $\{\varepsilon_{jk}\}$, with no need for additional support.

Besides, it is also necessary to remark that the system's configuration may change, *as it happens during evolution transition phases*, with no change in the system's structure $\{\varepsilon_{jk}\}$. Consider that the *state parameter h* relates only to standing states (temporary equilibrium states); thus, the equations [3.40], [3.42], [3.43], [3.48] and [3.49] do not apply to *transition phases*, in which the products $\{hP_j\}$ and $\{hQ_k\}$ shall be replaced by $\{Y_j\}$ and $\{X_k\}$, respectively, since the system's *base* (*i.e.,* $\{P_j\}$ together with $\{Q_k\}$.) changes from phase to phase. The mentioned replacements are possible thanks to the equations [3.37a] and [3.37b], which make the *structure potentials* depend on the system's structure $\{\varepsilon_{jk}\}$ only. In fact, by definition, during transition phases there is no equilibrium at all: These phases of the evolution involve continued changes both in the system's *base* and in the interaction distribution. A new state parameter h can be re-determined only if the sequence of the transition phases ends in a new standing state through a *transformation of the system's structure.*

As previously argued, state factor h is related to *base syntropy* S^*, which in turn depends on the features of the system's base. Considering also the range of the possible values for h (remember [3.35]), an appropriate theoretical interpretation of h is that this parameter indicates *the probability*

for the system *to lose its temporary equilibrium state.* In simpler terms, there are good reasons for assuming that *h* indicates the *degree of instability* of the system's equilibrium.

In analyzing the flow probability equations, the mathematical expression in the right-hand side of the equations may be viewed as consisting of three joint "sections". The first section is formed by state factor *h*, the second section by *random flow* probability P_{jk}^*, the third section by relation coefficient ε_{jk}. The three sections multiply each other: As a simple *factor*, state factor *h* - which is always less than 1 – diminishes probability P_{jk}^* *linearly*, whereas relation coefficient ε_{jk} modifies the same probability *exponentially* with the value of the interaction's intent.

Looking at the definitions [3.41] given for $\{P_j\}$ and $\{Q_k\}$ through the introduction of the *structure potentials*, the ratios of these to *h* do actually form two new probability distributions, in each of which the sum of the probabilities equals 1. Thus, each ratio, Y_j/h or X_k/h, seems just indicating how *stable* is the relevant probability P_j or Q_k in the system's standing state of *temporary equilibrium*: The greater its "instability" *h* the smaller the probability that the system's components remain in their present state of interaction sources and destinations.

Using different words, *state factor h* might also be considered as a *factor of uncertainty* concerning the assessment of either the interaction flow probability or of the interaction flow in the system's temporary equilibrium state described by the observer.

If this interpretation of parameter *h* is appropriate, then – as an obvious consequence – the *base syntropy S^** can simply be assumed to represent the system's *temporary degree of stability* since (remember the equations [3.32] and [3.33]) "instability" *h decreases exponentially with S^**.

Because of the range [3.35] of variability for *h*, one should conclude that there is *no state* of permanent stability for any recognizable system. Any *viable* system should be thought of as intrinsically unstable and therefore evolving.

3.4.4 *The meaning of "unstable equilibrium state"*

The "system's unstable equilibrium state" (*i.e., the original standing phase)* is a conjecture made by the observer before starting field surveys aimed at assessing the intensities of the specific interactions observed or guessed.

The observer supposes that, at least during the period in which the surveys are carried out, the overall state of the observed system does not change significantly, notwithstanding possible – and accounted for – oscillations of the measured quantities around average values that – in the observer's view – keep substantially constant.

It is a basic assumption, which rests on statistical criteria, measurement methods and approximations that the observer considers as appropriate to the study case.

On the one hand, the logic of the theory expounded here is an adequate support to the relevant undertaken applications only if the statistical methods and the measurement operations on the field are reasonable and correct.

On the other hand, the theory's equations provide the observer with the necessary instruments to check whether the findings of the surveys and the results of the measurement operations match the theoretical expectations in a satisfactory way.

The intrinsic instability of any evolving system entails inevitable problems in the measurement operations that regard the quantitative description of the systems observed. Substantially, the reliability of both survey findings and calculation results depends on the degree of *approximation* that the observer deems acceptable or useful to the purposes of the analysis.

Moreover, one major problem concerning the applications of this theory to real cases is that the "proper time" taken by the evolution of viable complex systems is not the "time" of Mechanics as measured by watches and astronomical calendars. The "evolution time" consists in the systems' "aging", which depends basically on the amount of *entropy* (disorder, energy dissipation, etc,) produced by the unceasing *gestation labor* inherent in the evolution processes. It is a point of philosophical trouble, for it seems actually impossible to "translate" aging processes in the precise, objective terms of "mechanical time". That is why also the watch or calendar *duration* of the system's equilibrium supposed by the observer might not correspond to any *organic stationary state* of the system under observation.

The preceding remarks intend to anticipate some insuperable difficulties encountered in using this theory for drafting *scheduled* events, which should instead be replaced by outlines of *probable* evolution trends, whose occurrence cannot be put in a precise correlation with astronomical dates. Who has some familiarity with meteorological models can better understand what I mean.

3.5 *Corollaries*

(1) In commenting on the constraint equation [3.15], I drew attention to the impossibility for the observer to assess the mean efficacy $\{u_{jk}\}$ of every interaction flow, for no objective data can in general be collected as to the individual "intents" that motivate the interactions, especially when the system's activators involve human beings. Nevertheless, the theory's equations formulated so far allow the analyst to assess the *intent matrix* $\{\mu_{jk}\}$ once completed the survey operations. The equations [3.0] and [3.33], in fact, enable one to write

[3.50]
$$\mu_{jk} = \ln\left(\frac{R_{jk}}{h}\right) = \ln\left(\frac{T_{jk}}{T_{jk}^*}\right) + S^* = \ln R_{jk} + 2\ln N - E^*, \qquad (\forall j, k).$$

in which, as seen with [2.39] and [2.22], *base entropy* can be expressed in two different ways as follows:

[3.51]
$$E^* = -\left(\sum P_j \ln P_j + \sum Q_k \ln Q_k\right) = 2\ln T - \frac{1}{T}\sum(O_i \ln O_i + D_i \ln D_i).$$

As to the analyst, the *matrix of intent* $\{\mu_{jk}\}$ is significant in itself, with no need to specify the measurement units that quantify the mean efficacy u_{jk} of every interaction flow and the corresponding value for coefficient λ.

Therefore, the equation [3.50] achieves one of the theory's announced goals, which is to express all the quantities involved by the theory in terms of interaction flows, or – equivalently – in terms of flow probabilities.

(2) From the equations [2.43] of Chapter 2, the following significant state parameter can be formulated:

[3.52]
$$F = \frac{S - S^*}{E^*} = \frac{E^* - E}{E^*} = 1 - \frac{E}{E^*} \quad ;$$

which indicates how much the system's activity *deviates* from that of the respective base configuration. The greater F with respect to its possible maximum the greater the intensity of the

"organized activity" that characterizes the system's interaction distribution. State parameter F is referred to as "**the system's strength**". Its possible maximum value is conditioned by the ratio E/E^*, considering that $E \leq E^*$ always (See also **Figure 2.1**, Page 34).

Constraint equation [3.15] defines a *mean efficacy u = U/T* associated with every interaction unit in the system, so that also a corresponding *mean intent $\mu = \lambda u$* can be defined by the product of λ with the same equation, to obtain

[3.53]
$$\lambda \sum_{j,k} u_{jk} P_{jk} = \sum_{j,k} \lambda u_{jk} P_{jk} = \lambda u = \mu .$$

In this equation each quantity $\mu_{jk} = \lambda u_{jk}$ can be replaced by the respective $\ln(R_{jk}/h)$, as it has been defined for the equations [3.29], page 53; in order to write (allowing also for the flow probability equation [3.35])

[3.54]
$$\mu = \sum_{j,k} P_{jk} \ln P_{jk} - \sum_{j,k} P_{jk} \ln P_{jk}^* - \ln h \sum_{j,k} P_{jk} .$$

Remembering the random flow probability equation [3.1], the second sum in the above equation can be written as

[3.55]
$$-\sum_{j,k} P_{jk} \ln P_{jk}^* = -\sum_{j,k} (P_{jk} \ln P_j + P_{jk} \ln Q_k) ;$$

from which, developing its right-hand side, one obtains

[3.56]
$$-\sum_{j,k} P_{jk} \ln P_{jk}^* = E^* . \quad \textbf{25}$$

Therefore, accounting for [3.32], equation [3.54] becomes

[3.57]
$$\mu = -E + E^* + \mu^* ,$$

or else, because of conjecture [3.32] and remembering [2.43], also

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25 To understand why this equation is true, consider the following example that regards a 3-component system. From the right-hand side of the equation [3.55] above:

$$\sum_{j,k=1}^{3} P_{jk} \ln P_j = \sum_{j=1}^{3} (P_{j1} \ln P_j + P_{j2} \ln P_j + P_{j3} \ln P_j) =$$

$$= P_{11} \ln P_1 + P_{12} \ln P_1 + P_{13} \ln P_1 + \qquad \rightarrow P_1 \ln P_1 +$$
$$+ P_{21} \ln P_2 + P_{22} \ln P_2 + P_{23} \ln P_2 + \qquad \rightarrow P_2 \ln P_2 +$$
$$+ P_{31} \ln P_3 + P_{32} \ln P_3 + P_{33} \ln P_3 \qquad \rightarrow P_3 \ln P_3 +$$

Analogously:
$$\sum_{j,k=1}^{3} P_{jk} \ln Q_k = \qquad \begin{array}{l} + \\ \rightarrow Q_1 \ln Q_1 + \\ Q_2 \ln Q_2 + \\ Q_3 \ln Q_3 = \rightarrow \quad -E_3^* , \end{array}$$

which represents the opposite of the *base entropy* equated by [3.56], where it is multiplied by –1. In this connection, remember how [3.51] defines E^*.

[3.58] $$\mu - \mu^* = \mu - S^* = E^* - E = S - S^* ,$$

which implies

[3.59] $$S = \mu = \lambda u \ .$$

That is: The system's syntropy (in its meaning of *degree of organization*) is directly proportional to the mean efficacy of the system's interaction unit.

(3) Besides, it is interesting to note that

[3.60] $$\sum_{j,k} P_{jk}^* \ln P_{jk}^* = \sum_{j,k} P_{jk} \ln P_{jk}^* = -E^* .$$

Despite its illogical appearance, this significant equation shows that the probability values of the system's flow distribution are subjected to an additional *hidden constraint*, which is implicit in the constraints expressed by the relations [3.15] to [3.17]. Such a condition can also be written as

[3.61] $$\sum_{j,k} (P_{jk} - P_{jk}^*) \ln P_{jk}^* = 0 .$$

3.6 *The "implicit flow": The external component's self-interaction*

The use of the whole set of the foregoing equations presupposes that *the system is closed*, which means that all the system's interaction flows are known. The theory has no possibility of viable application if the system is *not properly closed.*

However, the necessary baseline-survey, which shall be carried out to quantify the interactions relevant to the *original standing state* of the system, cannot detect the *self-interaction* that pertains to the *external component*. As previously pointed out, the *external component* is "the rest of the universe" *with respect to the main system* identified by the observer. Instead, the interactions between each component of the main system and the external component can be surveyed and quantified: Think – for example – of the export-import flows relative to the production sectors of any regional or national economic system.

This paragraph intends to show a possible procedure to "calculate" the "**implicit flow**" (*i.e.*, the *self-interaction* of the *external component*), *which is viewed as an implication of the activity characterizing the main system.* Intuitively, the main system's activity entails a specific complementary activity *inside* the *rest of the universe,* to the extent to which the latter interacts with the main system identified by the observer. Referring again to the example of a regional economic system, *the rest of the universe* does necessarily mobilize its *internal* economy *also* in consequence of its relationships with the *main system* under study. Several different examples are also possible, obviously.

The point to grasp is that the self-interaction *missed* by the baseline survey *is not* the whole self-interaction of "the rest of the universe", but *only* that part of the *inner* activity of the *external component* that is both connected with its relationship with the *main system* and of the same kind as that of the main system. For example, if the observer/analyst deals with vehicular traffic flows between settlements of activities (of which his system may consist), then **the relevant** *implicit flow*

involved *inside* the *external component* will also consist of vehicular traffic flows between settlements of activities "within the rest of the universe".

The self-interaction of the external component is "implicit" in the way in which the observer/analyst recognizes and describes the *main system* he has identified. This is a basic point to bear in mind while managing concepts relative to this theory.

Therefore, only after the determination of the *implicit flow* the theory's system can be considered as *closed* and ready for useful simulations.

3.6.1 *Calculation of the "implicit flow"*

In a view to proceed on in a clearer way, I deem it convenient to start soon from a numerical example.

Consider the following results of a hypothetical survey carried out to detect the interaction flow matrix of a four component system, in which T_{oo} stands for the missing *implicit flow*:

Four-component system - interaction flow matrix

Component labels	0	1	2	3	ORIGINS	
0	T_{oo}	15	12	16	43+T_{oo}	*Oo*
1	18	21	22	18	79	*O1*
[3.62] 2	7	4	16	9	36	*O2*
3	9	10	13	19	51	*O3*
DESTINATIONS	34+T_{oo}	50	63	62	209+T_{oo} =	
	Do	*D1*	*D2*	*D3*		*T*

The *implicit flow* T_{oo} is *the unknown* to be determined. Statistically, the interaction flows recorded in [3.62] are *probable flows*, which, according to the theory, are in general expressed by the equation [3.21], that is by

[3.63]
$$T_{jk} = h \frac{O_j D_k}{T} e^{\lambda u_{jk}}, \qquad (\forall j, k).$$

With respect to this formula, quantities O_j , D_k and T_{jk} are known parameters relevant to every flow except T_{oo}. The system's total interaction T can be determined only after calculating T_{oo}. Instead, an *usable* value for ratio h / T can be determined after assigning an *arbitrary* value to any one of the N^2 intents $\{\mu_{jk}\}$ of the system represented. To this purpose, consider that – whatever the measurement system adopted – it is always possible to assume the value of any *intent* as the *intent unit value*. One practical criterion may be adopting the smallest relation coefficient of the system as the exponential of the *intent unit*, which means to set the same exponential coefficient equal to $e = 2.7182818…$

Moreover, allowing for the formula [3.63], it is possible to write:

[3.64]
$$\frac{h}{T} e^{\mu_{jk}} = \frac{h}{T} \varepsilon_{jk} = \frac{T_{jk}}{O_j D_k} , \qquad (\forall j, k).$$

Clearly, irrespective of the ratio h/T, the smallest relation coefficient is given by the smallest ratio $\dfrac{T_{jk}}{O_j D_k}$. From the matrix of the *main system*, which concerns the components *1, 2* and *3* only, one obtains the minor matrix $\left\{\dfrac{h}{T}\varepsilon_{jk}\right\}$ as follows:

[3.65]

	1	*2*	*3*
1	5.3164×10^{-3}	4.4203×10^{-3}	3.6749×10^{-3}
2	2.2222×10^{-3}	7.0546×10^{-3}	4.0322×10^{-3}
3	3.9215×10^{-3}	4.0460×10^{-3}	6.0088×10^{-3}

The smallest element of this matrix is

$$\frac{h}{T}\varepsilon_{21}=2.2222\times10^{-3}\ .$$

Therefore, assuming $\mu_{12.}=1$ entails $\varepsilon_{21}= e^1 = 2.7182818$, by which the value of ratio $m.=.h./.T$ can be calculated accounting also for flow T_{21} :

[3.66]
$$m=\frac{h}{T}=\frac{T_{21}}{\varepsilon_{21}O_2 D_1}=\frac{4}{2.7182818\times36\times50}=8.1750988\times10^{-4}\ .$$

[Note, by the way, that the value found for *m* enables the analyst to calculate all the system's *intents* through the following obvious equations:

[3.67]
$$\mu_{jk}=\ln\left(\frac{T_{jk}}{O_j D_k m}\right)=\ln\left(\frac{T_{jk}}{T_{jk}^* h}\right)=\ln\left(\frac{R_{jk}}{h}\right),\qquad (\forall\, j,\, k),$$

the measurement system being established for $\{\mu_{jk}\}$ (and implicitly also for $\{u_{jk}\}$ and λ) by $\mu_{21}=1$, as per the analyst's decision].

Because of the range of the values of *h* and because of [3.66], it is evidently true that

[3.68]
$$h=mT<1\ .$$

This inequality establishes the upper limit to the value T_{oo} of the *implicit flow*, since, defining $T'=T-T_{oo}$ whence also $T=T'+T_{oo}$, and so replacing T in [3.68], one can write

[3.69]
$$T_{oo}<1/m-T'\ .$$

Then, from [3.66], $1/m=1223.2268$ and, from [3.62], $T'=209$; so that the upper limit to the value of the *implicit flow* is determined as

[3.70]
$$T_{oo}<1014.2268\ .$$

T_{oo} is subjected to a couple of additional constraints that are connected with the two different ways in which *base entropy* can be expressed: One way of the mentioned two is showed by the following expression derived from the equation [3.51]:

[3.71]
$$E^* = 2\ln T - \frac{1}{T}\sum_{i=0}^{N}\left(O_i \ln O_i + D_i \ln D_i\right) \ .$$

Therefore, set $O_o = O_o{}' + T_{oo}$ and $D_o = D_o{}' + T_{oo}$, it is possible to write (accounting also for $T = T'' + T_{oo}$):

$$E^* = 2\ln(T' + T_{oo}) -$$

[3.72]
$$-\frac{(O_o' + T_{oo})\ln(O_o' + T_{oo}) + (D_o' + T_{oo})\ln(D_o' + T_{oo}) + L}{T' + T_{oo}} \ ,$$

in which

[3.73]
$$L = \sum_{i\neq 0}^{N}\left(O_i \ln O_i + D_i \ln D_i\right).$$

The other way to express *base entropy* is provided by [3.34], by which – allowing for [3.66] – it is possible to write:

[3.74]
$$E^* = \ln(hN^2) = \ln(mN^2 T) = \ln(mN^2) + \ln(T' + T_{oo}).$$

After comparison of this equation with [3.72] above, one obtains the following function of T_{oo} equated to zero :

[3.75]
$$f(T_{oo}) = \frac{(O_o' + T_{oo})\ln(O_o' + T_{oo}) + (D_o' + T_{oo})\ln(D_o' + T_{oo}) + L}{T' + T_{oo}} -$$
$$-\ln(T' + T_{oo}) + \ln(N^2 m) = 0,$$

in which, when – for example – referred to the system's configuration [3.62], the known term is

[3.76]
$$\ln(N^2 m) = \ln(16\times 8.1750988\times 10^{-4}) = -4.3366588.$$

It can easily be proved that function $f(T_{oo})$, in the interval of the values for T_{oo} expressed by

[3.77]
$$0 \le T_{oo} < 1/m - T' \ ,$$

is continuously increasing with T_{oo}. The function is comprised between a lower extreme $f(T_{oo}) < 0$ and an upper extreme $f(1/m - T') > 0$ in such a way so as to make the Cartesian curve of $f(T_{oo})$ intersect the abscises once only. At the intersection, that is at $f(T_{oo}) = 0$, the value of T_{oo} at that point is the value searched for the *implicit flow*.

A rather quick demonstration concerning the uniqueness of the intersection of $f(T_{oo})$ with the abscises T_{oo} can be given as follows.

Should function $f(T_{oo})$, in its definition interval, intersect the abscises more than once, it should also be possible to determine at least one maximum or minimum value for the same function: This means it would be possible to obtain a significant solution for the following equation

[3.78]
$$\frac{df(T_{oo})}{dT_{oo}} = 0 \ .$$

For writing simplification purposes, let's adopt a formal change in the symbolism of the function's variables, as per the following definitions:

[3.79a] $$x = O_o' + T_{oo} \; ; \qquad (O_o' \le x < O_o' + \frac{1}{m} - T')$$

[3.79b] $$y = D_o' + T_{oo} \; ; \qquad (D_o' \le y < D_o' + \frac{1}{m} - T')$$

[3.79c] $$z = T' + T_{oo} \; ; \qquad (T' \le z < \frac{1}{m}) .$$

By replacement of the symbols of $f(T_{oo})$ with the just defined ones, the same function becomes

[3.80] $$g(x,y,z) \equiv f(T_{oo}) = \frac{x \ln x}{z} + \frac{y \ln y}{z} + \frac{L}{z} - \ln z + \ln(N^2 m) .$$

Equation [3.78] implies that the necessary condition for the existence of a point of maximum or minimum value for $f(T_{oo}) \equiv g(x,y,z)$ is also expressed by the following simultaneous equations:

[3.81] $$\frac{\partial g}{\partial x} = 0 \; ; \qquad \frac{\partial g}{\partial y} = 0 \; ; \qquad \frac{\partial g}{\partial z} = 0 \; .$$

Thus, after calculation of the above partial derivatives, one finds:

[3.82a] $$\frac{\partial g}{\partial x} = \frac{\ln x + 1}{z} = 0 ; \quad \text{whence} \quad x = O_o' + T_{oo} = O_o = e^{-1} .$$

[3.82b] $$\frac{\partial g}{\partial y} = \frac{\ln y + 1}{z} = 0 ; \quad \text{whence} \quad y = D_o' + T_{oo} = D_o = e^{-1} .$$

[3.82c] $$\frac{\partial g}{\partial z} = z + (x \ln x + y \ln y + L) = 0 ,$$

which, remembering the definitions given by [3.73] and [3.80], leads to the following *absurd assertion*:

[3.83] $$T = \sum_{i=0}^{N} \left(O_i \ln O_i + D_i \ln D_i \right) .$$

On the other hand, the results provided by [3.82a] and [3.82b] are also absurd, considering that such results would be true in general, irrespective of the size and configuration of the system. Consider also that these same results could analogously be obtained with reference to any other generic self-interaction $T_{jj} \ne T_{oo}$, should T_{jj} be the only unknown self-interaction to be determined for the system. Therefore**,** function $f(T_{oo})$ has neither points of *maximum* nor points of *minimum*, for the tangent to its Cartesian diagram is in no point parallel to the abscises. Moreover, if the appropriate implicit flow that satisfies the equation [3.75] is $T_{oo} > 0$, then $T_{oo} = 0$ implies $f(T_{oo}=0) < 0$, necessarily.

Instead, any other $T_{oo} > 0$ greater than that which satisfies the equation [3.75] renders $f(T_{oo}) > 0$.

Conclusion: As it was to prove, $f(T_{oo})$ is a continuous function that increases with T_{oo} ; the function is in general comprised between a negative lower extreme and a positive upper extreme.

Then, the Cartesian graph representing the function intersects the abscises in one point only, which corresponds to the searched value for the implicit flow T_{oo} .

It is now possible to proceed with the example exercise proposed through the hypothetical configuration [3.62] for which the missing "implicit flow" must be determined. Using the known data of that configuration, a graph of the relevant function $f(T_{oo})$ (*i.e.*, of the left hand side of the equation [3.75]) can easily be obtained by the aid of *Excel* software. It is initially convenient to build the graph using relatively large value intervals for T_{oo} with a view to identifying a suitable narrow interval of the independent variable within which the graph intersects the abscises. Subsequently, using much smaller value intervals for T_{oo} , and according to the desired degree of approximation, the value T_{oo} that zeros the function can easily be seized.

In the graphs that follow, the value intervals are initially fixed in 50 flow units; subsequently, in the second and third approximations, the intervals are 0.25 and 0.01 flow units, respectively. See **Figures 3.1** and **3.2**.

Figure 3.1

*DETERMINATION OF THE "IMPLICIT FLOW" BY EXCEL*

Variable T_{oo}
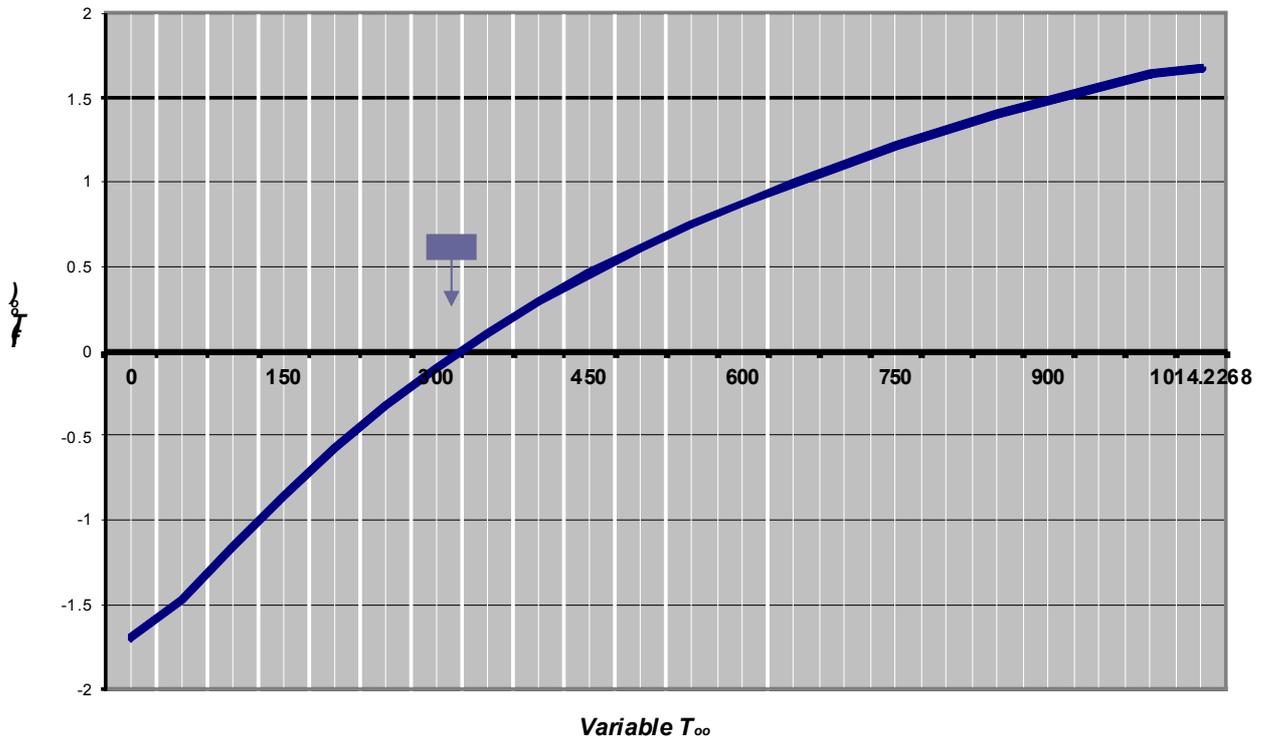
Figure 3.2

DETERMINATION OF THE "IMPLICIT FLOW" AT A MORE DETAILED SCALE
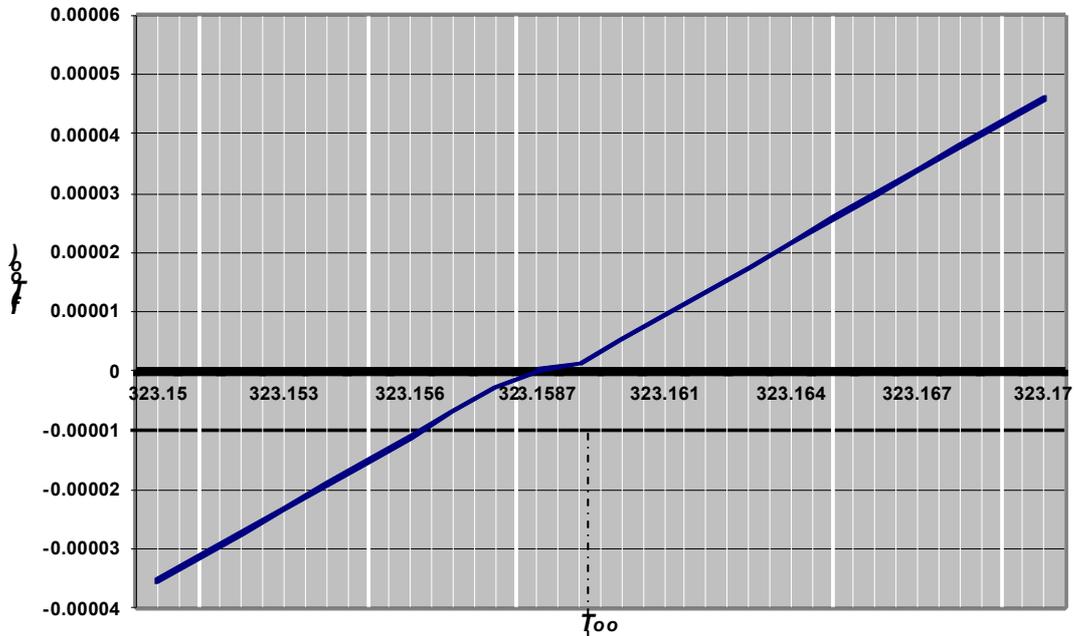
$$f(T_{oo}) = 0.\,000.000 \text{ at } T_{oo} = 323.1589$$

Thus, it may be stated that $f(T_{oo} = 323.1589) = 0.000\,000\,3537$ means *in practice* that $T_{oo} = 323.1589$ is the searched implicit flow relative to the system's configuration [3.62].[26]

One can soon observe that the implicit flow determined by this example exercise is noticeably larger than any other interaction flow of the system. As far as my experience in the field of socio-economic systems is concerned, that is the norm: The "implicit flow" determined in every application of the theory was found remarkably larger than any other interaction surveyed in the operations leading to the definition of the system. It is also a fact that responds to intuitive expectations and may justify my insistence in drawing attention to the importance of the "rest of the universe" (*i.e.,* of the system's *external component*) in representing any system to analyze. It is so understandable that the implicit flow plays a major role in the description of the system's evolution, as in most cases it is the principal contributor to the determination of the system's syntropy.

---

[26] It is worth noting that *Excel* software can always and easily be used for the quick determination of the "implicit flow", whatever the system's size, upon the very few data that are *in every case* necessary and certainly available for the definition of $f(T_{oo})$.

4. The system's evolution equations and parameters

4.1 *The standing configuration of the hypothesized system*

The determination of the "implicit flow" closes the system's configu-ration according to the set of interaction flows detected by the direct survey. Only after *closing* the system's configuration it is possible to proceed with the application of the theory.

Referring to the exercise example concerning the hypothesized four-component system introduced in the previous chapter (see [3.62]), the relevant closed standing configuration is now as follows:

Figure 4.1

THE SYSTEM'S *INTERACTION FLOW* ONFIGURATION					
$\{T_{jk}\}$					
Component labels	0	1	2	3	ORIGINS
0..	323.1587	15	12	16	366.1587	O_o
1..	18	21	22	18	79	O_1
2..	7	4	16	9	36	O_2
3..	9	10	13	19	51	O_3
DESTINATIONS	357.1587	50	63	62	**532.1587**
	D_o	D_1	D_2	D_3	T

in which T_{oo}= 323.1587 is the calculated *implicit flow.* By use of the definition [2.12], this flow configuration is translated into the relative standing *flow probability* configuration as follows:

Figure 4.2

	$\{P_{jk}\}$				
Component labels	0	1	2	3	Origin probability
0	0.607260	0.028187	0.022550	0.030066	0.688063	P_o
1	0.033824	0.039462	0.041341	0.033824	0.148452	P_1
2	0.013154	0.007517	0.030066	0.016912	0.067649	P_2
3	0.016912	0.018791	0.024429	0.035704	0.095836	P_3
Destination probability	0.671151	0.093957	0.118386	0.116507	**1.000000**
	Q_o	Q_1	Q_2	Q_3	*SUM*

Both configurations form the quantified image of the system as described in one of its supposed *standing states*; in particular, as it is described in that state of unstable temporary equilibrium which the observer/analyst considers as the system's *original state.*

It is also useful and interesting to evidence the relevant *base system* both in terms of flows and in terms of *flow probability*, as shown by the following **Figures 4.3** and **4.4:**

Figure 4.3

THE SYSTEM'S BASE CONFIGURATION					
(The base flow matrix)					
$\{T_{jk}^*\}$					
Component labels	0	1	2	3	ORIGINS
0	245.7479	3.,4032	43.3480	42.6599	366.1589 O_o
1	53.0209	7.4226	9.3525	9.2040	79 O_1
2	24.1614	3.3824	4.2619	4.1942	36 O_2
3	34.2287	4.7918	6.0377	5.9418	51 O_3
DESTINATIONS	357.1589	50.0000	63.0000	62.0000	**532.1589**
	D_o	D_1	D_2	D_3	T

Figure 4.4

THE SYSTEM'S BASE CONFIGURATION					
(The base flow probability matrix)					
		$\{P_{jk}^*\}$			*Origin*
Component labels	0	1	2	3	*probability*
0	0.461794	0.064648	0.081457	0.080164	0.688063 P_o^*
1	0.099634	0.013948	0.017575	0.017296	0.148452 P_1^*
2	0.045403	0.006356	0.008009	0.007882	0.067649 P_2^*
3	0.06432	0.00900	0.01135	0.011166	0.095836 P_3^*
Destination probability	0.671151	0.093957	0.118386	0.116507	**1.000000**
	Q_o^*	Q_1^*	Q_2^*	Q_3^*	*SUM*

It is now possible to determine all the system's relative *state parameters*:

[4.1]   Entropy potential $\quad H_4 = 2\ln N = \ln 16 = 2.7725887$

[4.2]   Entropy $= E = -\sum_{j,k} P_{jk} \ln P_{jk} = \ln T - \left(\frac{1}{T}\right) \sum_{j,k} T_{jk} \ln T_{jk} = 1.7041659$

[4.3]   Syntropy $= S = H - E = 1.0684229$

[4.4]   Base entropy $= E^* = \ln(hN^2) = -\sum_{j,k} P_{jk}^* \ln P_{jk}^* = 1.940283$

[4.5]   Base syntropy (or *stability*) $= S^* = H - E^* = 0.8323057$

[4.6]   State factor (or state *instability*) $= h = e^{-S^*} = mT = 0.4350452$

[4.7]   State strength $= F = 1 - E/E^* = 0.121692$;

   Particularly important is now the possibility of determining the complete set of the *intents* that characterize the structure of the system. Remembering equation [3.30] together with the definition given in [3.29] for "flow ratio", the system's matrix of intents $\left\{ \mu_{jk} = \ln\left(\dfrac{R_{jk}}{h} \right) \right\}$ is as follows:

Figure 4.5

THE SYSTEM'S STRUCTURE: THE *INTENTS*				
	$\{\mu_{jk}\}$			$h =$
Component labels	0	1	2	3	0.4350452
0	1.1061439	0.0022077	**-0.4520480**	**-0.1483652**	
1	0.7803529	1.8722995	1.6877078	1.5030374	
2	0.6659493	1	2.1551829	1.5958191	
3	0.6043910	1.5679843	1.5992369	1.9947268	

   From which also the respective *relation matrix* $\left\{ \varepsilon_{jk} = e^{\mu_{jk}} = \dfrac{R_{jk}}{h} \right\}$:

Figure 4.6

THE SYSTEM'S STRUCTURE: THE *RELATION COEFFICIENTS*				
	$\{\varepsilon_{jk}\}$			$h =$
Component labels	0	1	2	3	0.4350452
0	3.0226800	1.0022101	0.6363239	0.8621162	
1	2.1822423	6.5032332	5.4070722	4.4953225	
2	1.9463374	2.7182818	8.6294688	4.9323677	
3	1.8301373	4.7969694	4.9492541	7.3501950	

   The intents μ_{02} and μ_{03} in **Figure 4.5** offer the opportunity for a short comment on the meaning that the theory attaches to *negative values* expressed by such parameters. As seen, any "intent" is the product of a "mean efficacy" u_{jk} and Lagrangian constant λ, the latter being always a positive number. Therefore, any *negative* "intent" entails that the *efficacy* involved is necessarily negative too. The meaning of *negative efficacy* depends on the nature of the system considered: For example, if it is any kind of socio-economic system, negative efficacy may mean either *loss* or *aid* or *investment*. In other cases, instead, *negative efficacy* may simply indicate the mean "weight" of an interaction result that opposes the activators' expectations.

Concerning the determination of Lagrangian constant λ, the problem is negligible and its solution is generally unnecessary, as already remarked. However, the solution would in any case be immediate, once fixed the criterion for quantifying the elements of the *efficacy matrix* $\{u_{jk}\}$. For instance, if the interaction mean efficacy in our example system (after setting $\mu_{21} = \lambda u_{21} = 1$) is evaluated in Euros, then saying $u_{21} = 267.43 \, €$ implies that $\lambda = 1/267.43 = 0.0037396$ is the system's Lagrangian constant.

To complete the set of the same system's standing state parameters, the values for the *structure potentials* $\{X_i = hQ_i\}$ and $\{Y_i = hP_i\}$ - remembering the definitions [3.39] - are the following ones:

$$\{X_i\} \qquad\qquad\qquad \{Y_i\}$$

$$
\begin{array}{ll}
X_o = 0.2919810 & \qquad Y_o = 0.2993385 \\
X_1 = 0.0408755 & \qquad Y_1 = 0.0645833 \\
X_2 = 0.0515031 & \qquad Y_2 = 0.0294304 \\
X_3 = 0.0506856 & \qquad Y_3 = 0.0416930 \\
\end{array}
$$

[4.8]

$$\Sigma$$

Sum $\qquad 0.4350452 \quad = \quad h \quad = \quad 0.4350452$

One purpose of the phase parameters calculated and shown above is to exhibit also the numerical consistency of the theoretical definitions and equations presented in the previous chapters. As already pointed out, the quantified parameters defined so far regard the *original state* of the system, which the observer/analyst describes as that of a temporary equilibrium (*i.e.,* of an *original standing state*), prior to describing and discussing the system's possible evolution trends.

4.2 *The system's evolution*

The system described in the preceding chapters is identified through two aspects of its: The *configuration* and the *structure*, which have been kept distinct from each other, though closely interconnected by the equations expressed.

The terminological choice hints, through the relevant semantic connotation, also at the salient features of the two different aspects: The *configuration*, which refers to the system's *visible* and *measurable* aspect, *i.e.,* to the system as it appears to the observer; and the system's *structure*, which is guessed by the observer as a more stable underlying network of *causes* that motivate the rather unsteady configuration.

It is why the theory attaches less "inertia" to the system's configuration than to the system's structure, inasmuch as the configuration shows *irreversible* transformations of the system earlier than the relevant changes in the system's structure can be detected and assessed.

The reason for such an assumption is that the generation of *intentional* interactions depends on the activators' expectations associated with the *effects* of the actions undertaken. Interactions and evaluation of the relative effects is a process that takes time, this being more or less long according to the nature of the interactions considered. Moreover, the interaction effects are far from being all simultaneous, at variance with the detected interactions, which – as to each particular configuration – are instead surveyed or assessed with reference to the same unit of time.

The activators' evaluation of the quality of their interaction effects may either confirm or disappoint the respective expectations; in both cases, the *feed-back* from the interaction destinations may prime either tentative or irreversible – but not necessarily immediate – adjustments or major modifications to subsequent interactions.

In addressing *evolving systems*, the implicit obvious assumption is that the "phenomenon" observed and described is intrinsically unstable. More precisely, *it is a process*, which may or may not show states of *apparent* stability. Then, taken such an assumption for granted, the theory assumes also that the evolution process consists of sequences of tentative adjustments of the interactions to the impact of the system's overall activity on the activators' expectations. These expectations – in turn, albeit late with respect to the series of changes occurred in the interaction re-distributions – are corrected according to the effects of the precedent activity, with a view to keeping the system alive.

Considering the nature of the *intents* defined, it is supposed that the activators are constantly basing their activity on a perceived stability of the condition in which their expectations are to a various extent rooted, while stability seems instead escape the attempts to keep it permanently. Indeed, looking at the equations of the "intents" translated into functions of the system's stability, one clearly observes (remember [3.29]) that

$$[4.9] \qquad \mu_{jk} = \lambda u_{jk} = \frac{R_{jk}}{h} = R_{jk}e^{S^*} , \qquad (\forall j,k) ,$$

i.e., the expected *efficacy* of the interactions grows exponentially with the perceived stability S^* of the system.

Nevertheless, as already remarked, unexpected effects resulting from some of the interactions may cause relatively stable corrections to the interaction distribution *and* to the system's *base*, on which the stability rests.

The equations that describe the system's evolution move from this observation: Possible steady changes in some interactions may modify the system's base, particularly concerning the probability distribution of the interaction origins and destinations, with the subsequent modification of the entire configuration of the system. Such a modification is not a banal event, for it is necessarily connected with the whole activity of the system according to its internal links and constraints, as described by the theory's equations. Actually, any *minimal* alteration in the distribution of the base probabilities reflects on the whole system.

4.3 *The evolution process: A simulative representation*

The theoretical approach to the representation and simulation of the evolution process is based on the following six description criteria:

(i) Any apparent equilibrium state (*standing* state) of the system is the result of a precedent *process of transformation* that has involved *both* the interactions between the system's components *and* the intents of the activators.

(ii) Any standing state achieved by the system is subject to relatively slight oscillations about an average configuration, which the observer/analyst represents as the *original* image of the system.

(iii) Any new transformation process is primed by *any irreversible change* in the *base* of the *standing system;* the change entails a *redistribution sequence of the interaction flows*. Such a

redistribution sequence is referred to as the "sequence of *transition phases* of the *transformation cycle*", each transition phase being marked by specific *phase parameters*, partially different from the phase parameters that pertain to the standing states of the system.

(iv) The sequence of transition phases *develops with no change in the set of the activators' intents* (which constitute the system's *structure*) **until** the persistence of the activators' expectations does not jeopardize the system's survival. In other terms, to avoid the system's collapse, the need appears for a new settling down of the system, which implies, together with a new configuration of the system, also the *transformation* of the relevant set of *intents, i.e., a change also in the system's structure.*

(v) Every change in the system's structure concludes a transformation cycle and establishes a new *standing* equilibrium state, from which – sooner or later – a further transformation cycle will necessarily start. The only possible alternative to further transformation cycles is a transition process that inevitably leads to the disbandment of the system.

(vi) Important: The duration of any evolution process that includes more than *one* transformation *cannot* objectively be correlated with the astronomic conventional time. Instead, the duration of each *transition* phase *may* be correlated with the time unit used to quantify the interaction *flows* (see Paragraph 4.5 ahead). As to the theoretical stance, any evolution process is a particular *aging process*, proper to each particular system considered and described by specific phase parameters.

4.3.1 *Actual transition phases*

Bearing the foregoing notes in mind, remember the definitions

$$\sum_k T_{jk} = O_j \quad \text{as well as} \quad \sum_k P_{jk} = P_j, \qquad (\forall j)$$

referring to which and remembering also [3.47] and [3.44], after taking for known the supposed or observed modifications to the *origin semi-base* $\{O_j^{(1)}\}$, it is possible to write

[4.10]
$$Y_j \sum_k D_k^{(2)} \varepsilon_{jk} = O_j^{(1)}, \qquad (\forall j)$$

whence also

[4.10a]
$$\sum_k D_k^{(2)} \varepsilon_{jk} = \frac{O_j^{(1)}}{Y_j}, \qquad (\forall j)$$

together with

[4.11]
$$Y_j \sum_k Q_k^{(2)} \varepsilon_{jk} = P_j^{(1)}, \qquad (\forall j)$$

and

[4.11a]
$$\sum_k Q_k^{(2)} \varepsilon_{jk} = \frac{P_j^{(1)}}{Y_j}, \qquad (\forall j).$$

In these "transition equations" the superscript " $^{(1)}$ " indicates that the figures are the known quantities of the starting phase (*phase 1*) of the simulation, whereas the superscript " $^{(2)}$ " indicates

the quantities to be initially determined as unknowns. (The figures with no superscript are also known, for they pertain to the *original standing state*).

If the known figures used to start the simulation characterize one of the system's semi-bases, that particular modified semi-base may be referred to as the "**transition vector**".

The calculated solutions $\{D_j^{(2)}\}$ will subsequently go to form the known terms of the transition equations of *phase 2*. The unknowns of *phase 2* belong to the subsequent origin semi-base $\{O_j^{(3)}\}$, which, once determined, is in its turn used to form the set of the known terms of the subsequent transition equations in which $\{D_j^{(4)}\}$ are the unknowns to be determined. The determined $\{D_j^{(4)}\}$ are then used for the set of the known terms of the equations in the unknowns $\{O_j^{(5)}\}$ of the sub-sequent transition phase; and so forth.

Quite analogous sequences regard the $\{P_j^{(i)}\}$ and $\{Q_j^{(i+1)}\}$ of the equations [4.11] and [4.11a].

Obviously, the simulation of *the transformation cycle may in an alternative start* with known values given for the semi-base $\{D_k^{(1)}\}$ (or $\{Q_k^{(1)}\}$), and with the initial *simultaneous* flow equations

$$[4.12] \qquad\qquad \sum_j O_j^{(2)} \varepsilon_{jk} = \frac{D_k^{(1)}}{X_k} \quad , \qquad\qquad (\forall k)$$

or else with the simultaneous probability equations

$$[4.13] \qquad\qquad \sum_j P_j^{(2)} \varepsilon_{jk} = \frac{Q_k^{(1)}}{X_k} \quad , \qquad\qquad (\forall k)$$

and so on, in a "symmetrical" analogy with the equations [4.10] to [4.11a].

The relation coefficients $\{\varepsilon_{jk}\}$ remain unchanged by hypothesis up to the phase that concludes the transformation cycle, as these coefficients represent the system's original structure. Accordingly, along with the system's structure, also the structure potentials $\{Y_j\}$ and $\{X_k\}$ change only at the conclusion of each transformation cycle, as these "potentials" are steadily determined by the relation coefficients $\{\varepsilon_{jk}\}$ through the equations [3.37a] and [3.37b].

In this connection, it is once again worth pointing out that the equations [3.39], which – **in any standing configuration** of the system – connect the structure potentials with $\{P_j\}$ and $\{Q_k\}$ as well as with $\{O_j\}$ and $\{D_k\}$ through the relative state factor *h*, cannot pertain to transition phases, for during the transition process the system's configuration does change from phase to phase and the state factor *h* depends on a stability (remember $h = e^{-S^*}$) that is missing by the definition itself of "transition phase". (Actually, with reference to the equations [3.39] that define the "structure potentials", one may also suppose that in any transition *phase x* the products $Y_j = h^{(x)} P_j^{(x)}$ and $X_k = h^{(x)} Q_k^{(x)}$ remain constant, interpreting $h^{(x)}$ as the **latent** state factor of a *possible standing state* of the system if this could establish with the configuration provided by the transition *phase x*).

The sequence of transition phases cannot be very long; indeed, it is mostly short, since there is no mathematical theorem – as algebra teaches – that can secure *all positive* solutions to systems of simultaneous linear equations like the [4.10] to [4.12a].[27] That is why, after a few transition phases, also *negative* solutions appear that verify the simultaneous transition equations of a generic *phase z*.

Therefore, considering that both *negative interaction flows* and *negative interaction probabilities* are physically meaningless (at least in the context of this theory), the simulated sequence shall be

---

[27] One particular aspect of the *simultaneous* transition equations is that all the elements of the coefficient matrix $\{\varepsilon_{jk}\}$ and relevant transposed ${}^t\{\varepsilon_{jk}\}$ are positive quantities, albeit this fact is neither necessary nor sufficient condition to secure *all* positive solutions to the regarded equation systems.

stopped: Negative solutions to the simultaneous equations introduced above shall conventionally be considered as the indicators of the disbandment phase of the system. *Phase z is interpreted as the deadly stage in which the system collapses. Such a conclusion shall not be allowed for, if the simulation operator assumes that the system survives.*

The theoretical remedy to the system's disbandment (or collapse) is the simulated conversion of any transition phase that precedes the *disbandment phase z* into a *transformation phase*, in which the origin and destination semi-bases of two subsequent transition phases [for instance, *phase z-1* and *phase z-2*] are taken to form the *base* of a new *standing state* of the system, together with the relevant new structure and configuration. In most cases, the *transition phase z-1* is that which shows the best overall conditions for the system's transformation and survival.

4.3.2 *Back to the simulation exercise*

Let's go back to the example system of **Figures 4.3** and **4.4.**

Just to help understand the exercise, imagine that it represents the activity of an economic system in a given (original) year of activity, of which *Component 2* represents the *Industrial Sector*. At the end of that year, a few activators of *Component 2* realize that the response from the market makes it seem convenient to decide a slight increment in the output of their activities, so that the sales programmed for the following year bring the overall "homogenized" output of *Component 2* to 36.09 units (+0.25%) instead of the 36 units of the previous year. (Something analogous may obviously regard all the system's components; however, for simplifying this initial exercise, let the example focus only on the consequences of the decisions made by some activators of *Component 2*). Handling the figures of the example system, the first transition phase equations (refer to [4.10a] and [4.11a]) for the destination flows are:

$$O_j^{(1)}/Y_j$$

$$[4.14] \quad \begin{cases} 3.022680.D_o^{(2)}+1.002210.D_1^{(2)}+0.632324\,D_2^{(2)}+0.862116.D_3^{(2)}= 1223.2262 \\ 2.182242\,D_o^{(2)}+6.503233\,D_1^{(2)}+5.407072\,D_2^{(2)}+4.495323\,D_3^{(2)}= 1223.2264 \\ 1.946337\,D_o^{(2)}+2.718282\,D_1^{(2)}+8.629469\,D_2^{(2)}+4.932368\,D_3^{(2)}= \mathbf{1226.2830} \\ 1.830137\,D_o^{(2)}+4.796969\,D_1^{(2)}+4.949254.D_2^{(2)}+7.350195\,D_3^{(2)}= 1223.2269. \end{cases}$$

$T^1 = \mathbf{532.1587+0.09 = 532.2487}$ being the new hypothesized total interaction flow, the relative equations for the destination probabilities are:

$$P_j^{(1)}/Y_j$$

$$[4.14a] \quad \begin{cases} 3.022680\,Q_o^{(2)}+1.002210\,Q_1^{(2)}+0.632324\,Q_2^{(2)}+0.862116\,Q_3^{(2)}= 2.2982243 \\ 2.182242\,Q_o^{(2)}+6.503233\,Q_1^{(2)}+5.407072\,Q_2^{(2)}+4.495323.Q_3^{(2)}= 2.2982257 \\ 1.946337\,Q_o^{(2)}+2.718282\,Q_1^{(2)}+8.629469\,Q_2^{(2)}+4.932368\,Q_3^{(2)}= \mathbf{2.2691061} \\ 1.830137\,Q_o^{(2)}+4.796969\,Q_1^{(2)}+4.949254\,Q_2^{(2)}+7.350195\,Q_3^{(2)}= 2.2982275 \end{cases}$$

It can be observed that the known terms of these equations are not exactly equal to each other (as it should be - according to the equations [3.37] - if the system were instead in its original standing state), because the original equilibrium has been broken by the alteration made to $O_1^{(1)}$.

Considering that the alteration consists in 0.000169 of the system's overall activity, also the differences between most of the same known terms are only visible around their sixth significant digit.

The solutions $\{D_k^{(2)}\}$ and $\{Q_k^{(2)}\}$ to the equation systems [4.14] and [4.14a], respectively, go to compose the known terms of the subsequent equation systems in the unknowns $\{O_j^{(3)}\}$ and $\{P_j^{(3)}\}$, respectively, of *transition phase 3*; and so on, as per the precedent description given for the *transition process*. To note: the relation coefficient matrix of every *odd phase* equations is the *transposed matrix* of the preceding *even phase* equation system.

Due to the impossibility of using any "state factor" during the transition process, the configurations of the transition phases are determined by use of the "semi-canonical" equations introduced in Paragraph 3.3.2, namely by use of the equations [3.44] and [3.45] as follows:

[4.15] $$P_{jk}^{(i+1)} = Q_k^{(i+1)} Y_j \varepsilon_{jk}, \qquad (\forall i,j,k)$$

and

[4.15a] $$P_{jk}^{(i+2)} = P_j^{(i+2)} X_k \varepsilon_{jk}, \qquad (\forall i,j,k)$$

[4.14b] $$P_{jk}^{(i+3)} = Y_j Q_k^{(i+3)} \varepsilon_{jk}, \qquad (\forall i,j,k)$$

and so forth.

Consider also the equations $T_{jk}^{(i)} = P_{jk}^{(i)} T$, $(\forall i,j,k)$, if the total flow T^1 is supposed to remain constant during the transition process.

After carrying out the required calculations, one observes that there are negative solutions to the simultaneous equations relevant to the transition *phase 5*. Therefore, $z = 5$; and the simulation shall be stopped at the *phase z-1 = 4* In this context, any "*phase z-1*" will be referred to as the "**agony phase**" of the transition process.

The *interaction flow* configuration of the *phase 4* is as follows:

$$\{T_{jk}^{(4)}\}$$

Component labels	0	1	2	3	ORIGINS	
0	326.26	10.11	16.16	13.7	**366.23**	O_o
1	18.17	14.16	29.63	15.41	**77.37**	O_1
2	7.07	2.7	21.55	7.71	**39.03**	O_2
3	9.09	6.74	17.51	16.27	**49.61**	O_3
DESTINATIONS	**360.59**	**33.71**	**84.85**	**53.09**	**532.24**	
	D_o	D_1	D_2	D_3	T	

[4.16]

and the respective *interaction probability* configuration is given by

$$\{P_{jk}^{(4)}\}$$

Component labels	0	1	2	3		
0	0.612994	0.018995	0.030362	0.025740	**0.688092**	$P_o^{(3)}$
1	0.034139	0.026605	0.055670	0.028953	**0.145367**	$P_1^{(3)}$
2	0.013283	0.005073	0.040489	0.014486	**0.073332**	$P_2^{(3)}$
3	0.017079	0.012663	0.032899	0.030569	**0.093210**	$P_3^{(3)}$
	0.677495	**0.063336**	**0.159421**	**0.099748**	**1.000000**	
	$Q_o^{(4)}$	$Q_1^{(4)}$	$Q_2^{(4)}$	$Q_3^{(4)}$	*Sum*	

[4.16a]

Actually, the *phase 4* in this example of transition process is also the most convenient phase for the transformation of the system, since the transformation would bring a remarkable improvement in the system's organization, as shortly discussed in the next Paragraph 4.3.3.

The configuration of the transition *phase 4*, once transformed into that of a new standing state, shows a relatively high increase in the system's syntropy with a corresponding collapse of entropy, plus a set of parameters (some of them not yet introduced; see subsequent Paragraph 4.5) apt to synthesize and represent the set of the opportunities that (presumably) induce the systems' activators to modify their expectations.

The transformation from a *transition phase* into the corresponding latent *standing state* requires the transformation of the system's structure. Therefore, a new *standing state* for the system – seized the favourable opportunity represented in the example by the configuration [4.16] – is possible only if the hypothesized changes in the activators' expectations make the system's original relation coefficients $\{\varepsilon_{jk}\}$ become $\{\varepsilon^1_{jk}\}$; so as to write (remembering the "canonical" and "semi-canonical" equations [3.42] and [4.15]), the following equations for the new *standing state configuration*:

$$[4.17] \qquad P^1_{jk} = \frac{Y^1_j X^1_k}{h_1} \varepsilon^1_{jk} \equiv P^{(4)}_{jk} = Q^{(4)}_k Y_j \varepsilon_{jk} \quad , \qquad (\forall j,k)$$

in which the *index* or the *superscript* "1" marks the quantities pertaining to the new *standing state*, as achieved at the conclusion of the **first** *transformation cycle*, while the new state factor h_1 is determined accordingly. The transformed relation matrix $\{\varepsilon^1_{jk}\}$ is obtained through the conditions imposed by the new standing state configuration $\{P^1_{jk}\}$, *i.e.*, by the following conditions

$$[4.18] \qquad P^1_{jk} = Q^1_k Y^1 \varepsilon^1_{jk} \equiv P^{(3)}_{jk} = Q^{(4)}_k Y_j \, \varepsilon_{jk} \,, \qquad (\forall j,k)$$

from which also

$$[4.19] \qquad \varepsilon^1_{jk} = \frac{Q^{(4)}_k Y_j}{Q^1_k Y^1_j} \varepsilon_{jk} = \frac{Y_j}{Y^1_j} \varepsilon_{jk} \,, \qquad (\forall j,k)$$

as it is $\{Q^{(4)}_k \equiv Q^1_k\}$ because of the conversion of the semi-base $\{Q^{(4)}_k\}$ into the relevant semi-base $\{Q^1_k\}$ of the new standing state.

In fact, the *configuration* of the *new standing state* at the conclusion of the *first transformation cycle* coincides with the [4.16] of the *agony phase 4*. Then, the procedure for the *transformation* makes use of the semi-bases $\{O^{(3)}_j\}$ and $\{D^{(4)}_k\}$, or – equivalently – $\{P^{(3)}_j\}$ and $\{Q^{(4)}_k\}$, to calculate the stability S_1^* inherent in the new standing configuration, whose base is formed by the semi-bases of the transition *phase 4*, thus obtaining also the *state factor* $h_1 = e^{-S_1^*}$ of the new *standing state*. In this case, the calculation of S_1^* gives:

$$[4.20] \qquad S_1^* = 2\ln 4 - \left(\sum_i P^{(3)}_i \ln P^{(3)}_i + \sum_i Q^{(4)}_i \ln Q^{(4)}_i \right) = 0.8610345 \ .$$

Therefore,

$$[4.21] \qquad h_1 = e^{-S_1^*} = e^{-0.8610345} = 0.4227245 .$$

Then, it is also possible to determine the structure potentials pertaining to this standing state by use of the equations [3.39], thus obtaining the transformed structure potential matrices $\{X_i^1 = h_1 Q_i^{(4)}\}$ and $\{Y_i^1 = h_1 P_i^{(3)}\}$, *i.e.,* explicitly:

	$\{X_i^1\}$			$\{Y_i^1\}$
	0	0.2863938		0	0.2908733
	1	0.0267737		1	0.0614501
[4.22]	2	0.0673910		2	0.0309990
	3	0.0421660		3	0.0394021

Σ

$$0.4227245 \; = \; h_1 \; = \; 0.4227245$$

Allowing for the foregoing considerations, the *new relation matrix* pertaining to the newly achieved standing state is calculated through the equations [4.19] as follows:

THE **RELATION** MATRIX $\{\varepsilon_{jk}^1\}$ OF THE NEW STANDIG STATE

Component labels	0	1	2	3
0	3.1106483	0.2225231	0.0643828	0.1235734
1	10.630239	6.8348168	2.5896145	3.0500116
2	18.7945625	**5.6632545**	8.1927757	6.6339184
3	13.9035946	7.8626305	3.6967160	7.7775510

[4.23]

with the respective *intent matrix*:

THE **INTENT** MATRIX $\{\mu_{jk}^1 = \ln \varepsilon_{jk}^1\}$ OF THE NEW STANDING STATE

Component labels	0	1	2	3
0	1.1348312	**-1.5027246**	**-2.7429087**	**-2.0909196**
1	2.3637027	1.9220297	0.9515090	1.1151454
2	2.9335676	**1.7339987**	2.1032528	1.8921956
3	2.6321474	2.0621212	1.3074449	2.0512415

[4.24]

The configuration of the *new standing state* at the conclusion of the *first transformation cycle* – in terms of both *flow and interaction probability re-distribution* – is obviously that shown by [4.16] and [4.16a], respectively, as provided by the *agony phase 4*.

It is worth stressing again that the determination of a new structure for the system at the completion of every transition/transformation cycle is necessary both to keep the system in existence and to allow it to start further possible transformation cycles, since, through the previous structure, the system would have not survived beyond the *agony phase* of its transition process: The subsequent *transition phase z* would have been that of the system's disbandment.

4.3.3 *A few further comments on the example exercise*

The schematic simulation example conducted above is mainly aimed at proving the high sensitivity of the represented complex system, also when very minor *irreversible* changes intervene in its configuration. The only alteration made to the behaviour of Component 2, whose activity is initially incremented with just a quarter of percent point, leads to significant changes in the system's interaction distribution at the completion of the first transformation cycle, as shown by the differences between the new standing state configuration and the original one. In particular, the same Component 2 shows a remarkable increment in its own activity at the expenses of the other components.

In the example exercise, the choice of the *agony phase* for the system's transformation is associated with the noticeable increment in both the system's syntropy (which the theory interprets as degree of overall effectiveness) and stability, which alone, in case of *actual* sensitivity analysis, might be a justifiable planning choice for the decision makers.

Indeed, assuming the configuration [4.16] as the **transformation phase**, *i.e.,* as the phase of the transition process that makes its configuration become the configuration of a *new standing state*, one observes, for example, that the system's syntropy raises from the original $S = 1.068423$ to $S_1 = 1.102704$ in the new standing state, (+3.2%); entropy $E = 1.7041659$ falls to $E_1 = 1.669885$, while the system's stability also improves remarkably from $S^* = 0.832306$ to $S_1^* = 0.861034$, (+3.44%).

Not to omit the consolidation of the system's activity represented by the associated *phase strength* $F^1 = 1 - E_1/E_1^* = 0.1264238$, with its 3.89% increment (refer to [4.7]).

The increase in the number and in the absolute values of the negative intents relevant to the transaction between the *external component* and the other components of the system might in this case be interpreted as an increment in the aids and/or investments made by foreign activators in the production sectors of the *main system*. Due to the overall improvement showed by the system's newly achieved state, it would seem less credible that the same negative figures have to be interpreted as *increased losses* associated with the relevant transactions. This comment, however, is just a further example of how the simulation operator could be led to interpret (*possibly* but *not* necessarily) the results of the represented evolution.

A few additional phase parameters, which will later be proposed, would help planners to make more appropriate interpretations and decisions, according to the specific issues inherent in the study subject.

It should go without saying that, in simulating *possible* evolutions of the system, one may in various ways imagine (or detect) changes in the system's base; which means the possibility of *intervening* in the simulated "behaviour" of more than one component, *either* concerning the origin semi-base (along with the origin *probability* semi-base) *or* the destination semi-base (along with the destination *probability* semi-base), *or even* concerning both semi-bases. In the latter case, the *known alterations* (to be introduced only in starting the simulation) shall *initially* be transferred to the known term side of the simultaneous transition equations regarded, obviously letting the same semi-bases change automatically during the transition phases sub-sequent to the first one, according to the calculation process.

For example, if in the previous simulation exercise one also assumes that the quantity $D_1^{(2)}$ is known (along with the corresponding probability $Q_1^{(2)}$, obviously), then, after elimination of one unnecessary equation (say the second one), the system [4.14] would become

$$[4.14'] \quad \begin{cases} 3.022680\,D_o^{(2)} + 0.632324\,D_2^{(2)} + 0.862116\,D_3^{(2)} = 1223.2262 - \mathbf{1.002210}.D_1^{(2)} \\ 1.946337\,D_o^{(2)} + 8.629469\,D_2^{(2)} + 4.932368\,D_3^{(2)} = \mathbf{1226.2830} - \mathbf{2.718282}.D_1^{(2)} \\ 1.830137\,D_o^{(2)} + 4.949254.D_2^{(2)} + 7.350195\,D_3^{(2)} = 1223.2269 - \mathbf{4.796969}.D_1^{(2)} \ . \end{cases}$$

Analogous change shall clearly be made in the probability equations [4.14a].

However, there is to expect that the transition cycle could rapidly shorten when initial alterations *to both semi-bases* occur at once: It is the consequence of *exceeding constraints* to the system's change. It might happen that even the first transition phase be affected by "negative solutions", which means that the tried modification to the system's base is *impossible*.

The system's high sensitivity, due to the close interdependency that binds the system's activities to each other, does in general suggest – as also experience teaches – to operate evolution simulations through a longer series of transformation cycles, each started by relatively small changes made in the system's base in each of the subsequent standing states achieved.**[28]**

In carrying on evolution simulations, one should always allow for the observed evolution pace proper to the system considered. In this connection, the time unit adopted to quantify the interaction flows is the reference parameter for an appropriate sizing of the amount of each "artificial" modification made in the system's base for simulation purposes.

Whatever the measurement system adopted, if – for example – 100 out of 10000 total interactions is on an average the number of interaction units involved during one time unit of the system's evolution process observed, a *simulated change* in the system's base exceeding 100 interaction units per time unit *might* subject the system's "physiology" to an unbearable stress, as resulting in immediate negative solutions provided by the very first transition equations.

4.3.4 *Virtual transition phases*

The transition phases addressed so far are denoted "**actual phases**", to mean that they represent *probable future* evolution stages occurring in consequence of *given irreversible* changes in the system's base. In simpler terms, the "given" changes are supposed to be the *cause* of the simulated *future* evolution process. As already remarked, further transformation cycles can develop in consequence of further changes occurring in the base of every new *standing state* of the system. In principle, the series of subsequent *transformation cycles* cannot be limited, and the simulation of the evolution process may develop indefinitely.

In this theoretical context, it is necessary to pre-suppose also an evolution link between the *original configuration* of the system and *the first actual transition phase*, inasmuch as this *simulated first* transition – determined through *given* changes – must logically be thought of as belonging to a *transition process already in progress,* which did start from the *original* standing state because of *previous unknown changes* in the *original* configuration.

The supposed transition phases that precede the *first actual* simulated transition phase are referred to as "**the virtual phases"** of the simulated process; such phases simulate the *past* of the *first actual* transition phase, as the latter is supposed to be the "logical" consequence of its own *past*.

The point to grasp is that it is actually impossible to seize, recognize, isolate the *"objective"* original causes that lead one to observe (or imagine) and "give" the precisely quantified and irreversible changes necessary to determine the *first actual transition phase* of every sub-sequent transformation cycle.

A major limit of this theory is that it can neither formulate nor know any "spontaneous decision" made by the simulated system itself as to its own "choice" regarding its evolution path towards either survival or disbandment. "Choices" of the kind can possibly be simulated only through

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[28] No need to say that simulations of this kind, which usually involve a huge amount of numerical calculation, can only be performed by means of appropriate computerized procedures.

relatively arbitrary constraints – either through additional hypotheses or recorded observations – imposed by the simulation operator to the features of his representation.

The foregoing considerations entail that the simulated transition process shall be imagined as already in progress, irrespective of the phase that the simulation operator has *conventionally* established to be the "initial" one.

That is why another particular transition phase, which may be dubbed "*phase 0 *", can also be thought of, whose configuration (dealing for the moment only with the interaction probabilities) shares one of its two semi-bases with the configuration of the *initial phase*, the probability elements of which are expressed by the following N^2 semi-canonical equations:

[4.25] $$P_{jk}^{(1)} = P_j^{(1)} X_k \varepsilon_{jk} \qquad\qquad (\forall j, k)$$

As seen, this configuration cannot coincide with the *original* one, because - should this *kind* of semi-canonical equations regard the *original standing state* - the respective known terms would include the original matrix $\{P_j\}$, instead of $\{P_j^{(1)}\}$, the latter being – in the exemplification exercise - the modified semi-base (*i.e.,* the *transition vector*) of the *first actual* transition phase or *phase 1*.

It is therefore necessary to pay attention to the use of the superscripts that identify the *phases* of the simulation: The summations of the equations [4.25] with respect to the index "*k*" provide the elements of the *transition vector* $\{P_j^{(1)}\}$, whereas the summations of the same equations with respect to the index "*j*" provide the elements of the other semi-base $\{Q_k^{(0)}\}$ that belongs *both* to the *initial phase* and – symmetrically – to the *first virtual phase 0*, according to the following equations:

[4.26] $$\sum_k P_{jk}^{(1)} = P_j^{(1)} \sum_k X_k \varepsilon_{jk} = P_j^{(1)} \qquad\qquad (\forall j)^{\,3}$$

and

[4.27] $$\sum_j P_{jk}^{(1)} = X_k \sum_j P_j^{(1)} \varepsilon_{jk} = Q_k^{(0)} \qquad\qquad (\forall j)^{\,29}.$$

To be more precise, this particular semi-base $\{Q_k^{(0)}\}$ is the *virtual side* (to be calculated because unknown) of the *first actual phase* of the simulation, for the other *given* $\{P_j^{(1)}\}$ semi-base of the same *phase* is the actual *transition vector* that starts the sequence of the *actual phases*.

The features that distinguish the *initial phase*, *i.e.,* the "start phase" or *phase* 1, from the phase dubbed "*phase zero*", are obviously a matter of *conventional definitions* only; what really matters is to make the definitions unambiguous and clearly usable for practical purposes.[30]

Indeed, the probability matrix $\{P_{jk}^{(1)}\}$, whose elements are given by the equations [4.25], belongs entirely to the *actual phase 1* of the simulation, **though** the *destination probability semi-base* $\{Q_k^{(0)}\}$ of the same *phase 1* configuration belongs to the *phase 0.*

As a conclusion, one *virtual semi-base characterizes the configuration of the first actual transition phase*, always.

The elements that configure the *virtual phase 0* are expressed by the following N^2 semi-canonical equations:

---

[29] Remember [3.37b] and [4.13].
[30] Because of possible confusion, not only is it important to bear in mind the distinction between the *initial phase* (that is the *phase 1* or the *first actual phase*) and the *phase 0*, but also the fundamental distinction between both such phases and the **original** *standing state*.

[4.28]
$$P_{jk}^{(0)} = Y_j Q_k^{(0)} \varepsilon_{jk} \qquad\qquad (\forall j,k)$$

whose factors $\{Q_k^{(0)}\}$ are previously calculated by the equations [4.27].

Thanks to the configuration $\{P_{jk}^{(0)}\}$ determined through the equations [4.28], it is now possible – by an analogy with [4.27] – to determine also the other semi-base of the *virtual phase 0, i.e.,* $\{P_j^{(-1)}\}$, through the following semi-canonical equations (refer to [3.47]):

[4.29]
$$\sum_k P_{jk}^{(0)} = Y_j \sum_j Q_k^{(0)} \varepsilon_{jk} = P_j^{(-1)} \qquad\qquad (\forall j),$$

which will then allow one to write also

[4.30]
$$\sum_j P_{jk}^{(0)} = X_k \sum_j P_j^{(-1)} \varepsilon_{jk} = Q_k^{(-2)} \qquad\qquad (\forall k)$$

as well as

[4.31]
$$P_{jk}^{(-1)} = Y_j Q_k^{(-2)} \varepsilon_{jk} \qquad\qquad (\forall j,k),$$

and so on.

Thus, as it is now easy to guess, a "reverse" sequence of *virtual transition* equations has been initiated. This sequence represents a sort of mathematical "travel" through the probable *past* transition phases that have "spontaneously" sprung from the *original standing state*, virtually leading the system to the changes - either hypothesized or observed - which start the sub-sequent *actual* evolution sequence.

The *virtual configurations,* whose sequence is generated by the reverse chain of the *virtual transition equations* initiated by [4.26] to [4.31], tend **asymptotically** to coincide with the configuration of the *original standing state.* Such a kind of "reverse mathematical travel" through the *virtual phases* might sometimes (for example, when the *given* changes to any known previous original configuration are not hypothesized but observed) be useful *to try* a better understanding of where and how the very initial imperceptible and irreversible alterations to the original configuration took place.

(**Figure 4.7** in the next page is a graphic scheme that illustrates the development line of an evolution cycle, from the system's original standing state up to the *actual agony phase* that transforms into a new *standing* equilibrium state).

Figure 4.7

GRAPH SCHEMATIZING ONE OF THE SYSTEM'S TRANSFORMATION CYCLES
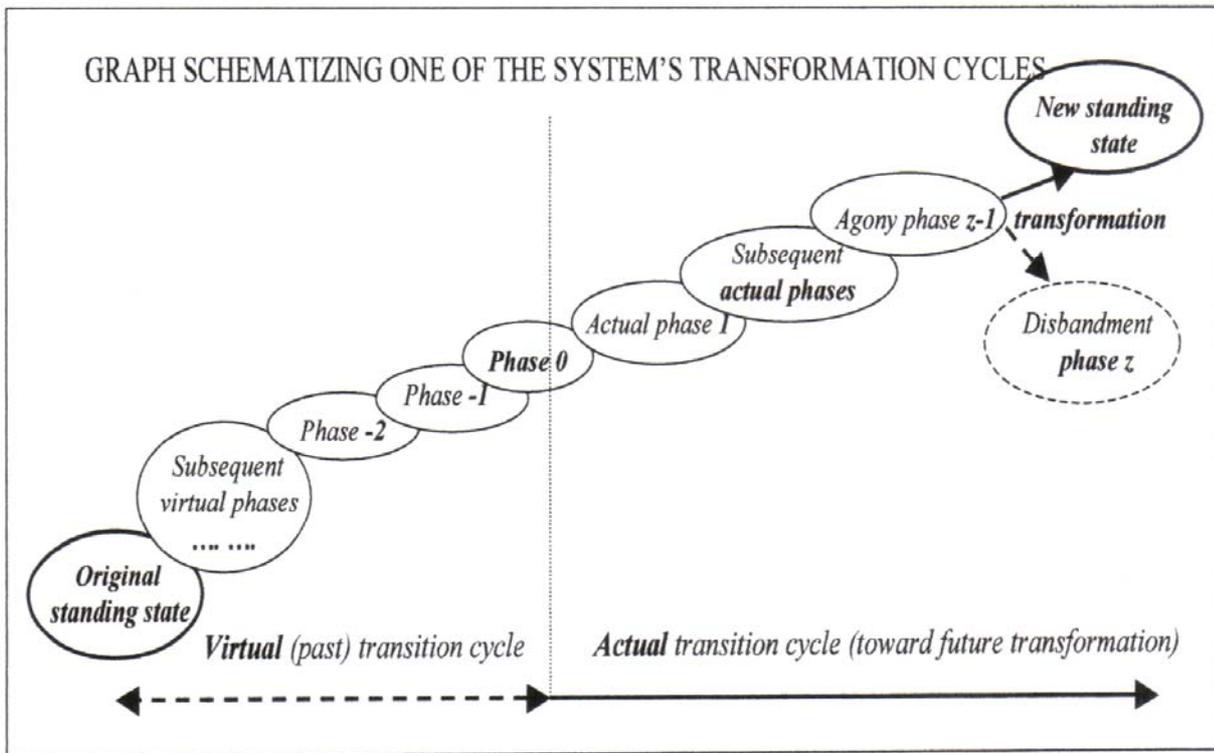
Figure 4.8

THE SYSTEM'S INTERACTION FLOW CONFIGURATIONS IN VARIOUS PHASES						
			Tjk			
Component labels	*0*	*1*	*2*	*3*	*ORIGINS*	
New standing phase flows	326.26	10.11	16.16	13.7	*366.23*	
Original flows "0".	323.1589	15	12	16	366.1589	O_o
Flows at virtual phase –9	323.21	15	12	16	366.21	
New standing phase flows	18.17	14.16	29.63	15.41	*77.37*	
Original flows "1".	18	21	22	18	79	O_1
Flows at virtual phase –9.	18.00	21.00	22.00	18.00	79.00	
New standing phase flows	7,07	2,7	21.55	7.71	39.03	
Original flows "2".	7	4	16	9	36	O_2
Flows at virtual phase –9	7.00	4.00	16.00	9.00	36.00	
New standing phase flows	9.09	6.74	17.51	16.27	49.61	
Original flows "3"	9	10	13	19	51	O_3
Flows at virtual phase –9.	9.00	10.00	13.00	19.00	51.00	
New standing destinations	360.59	33.71	84.85	53.09	532.24	
Original destinations	357.1587	50	63	62	532.1587	
Destinations in phase –9	357.22	50.01	63.01	62.01	532.25	
DESTINATIONS	D_o	*D1*	*D2*	*D3*	*T*	
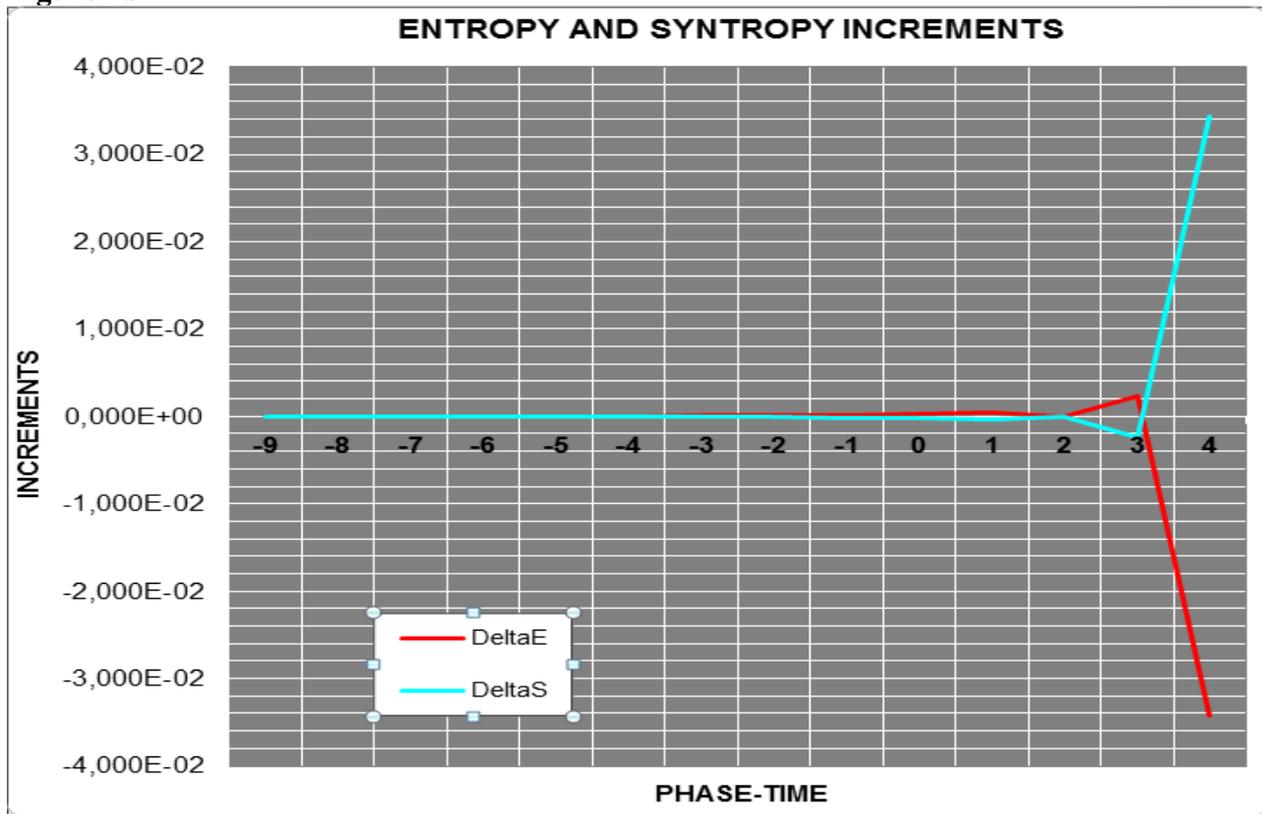
Figure 4.9

Figure 4.9a

PHASE	Original value E 1.7041659	Entropy Increments ΔE	Original value S 1.0684229	Syntropy Increments ΔS
-9		$1.900 \cdot 10^{-06}$		$-1.822 \cdot 10^{-06}$
-8		$1.010 \cdot 10^{-05}$		$-1.018 \cdot 10^{-05}$
-7		$1.410 \cdot 10^{-05}$		$-1.418 \cdot 10^{-05}$
-6		$2.110 \cdot 10^{-05}$		$-2.118 \cdot 10^{-05}$
-5	Virtual phases	$3.110 \cdot 10^{-05}$	Virtual phases	$-3.118 \cdot 10^{-05}$
-4		$4.710 \cdot 10^{-05}$		$-4.718 \cdot 10^{-05}$
-3		$6.910 \cdot 10^{-05}$		$-6.918 \cdot 10^{-05}$
-2		$1.031 \cdot 10^{-04}$		$-1.032 \cdot 10^{-04}$
-1		$1.471 \cdot 10^{-04}$		$-1.472 \cdot 10^{-04}$
0		$2.171 \cdot 10^{-04}$		$-2.172 \cdot 10^{-04}$
1		$3.821 \cdot 10^{-04}$		$-3.822 \cdot 10^{-04}$
2		$-1.390 \cdot 10^{-05}$		$1.382 \cdot 10^{-05}$
3	Actual phases	$2.384 \cdot 10^{-03}$	Actual phases	$-2.384 \cdot 10^{-03}$
4		$-3.428 \cdot 10^{-02}$		$3.428 \cdot 10^{-02}$

The table of **Figure 4.8** displays a comparison between the *interaction flows* calculated for the configuration of the *new standing state* with the configuration of the *original standing state* and with the configuration calculated for the *virtual phase −9*. It can be noted that the latter – albeit far from being the last possible *virtual configuration* – shows that most figures are already coincident

at the second decimal number with the interaction flows of the original configuration, to which all the figures of the virtual phases tend asymptotically.

   Figures 4.9 and **4.9b** are graphic representations of the transformation cycle in terms of entropy and syntropy *increments* with respect to the values of entropy and syntropy inherent in the original standing state, while **Figure 4.9a** lists the sequence of the increments illustrated by the graphs.

Figure 4.9b
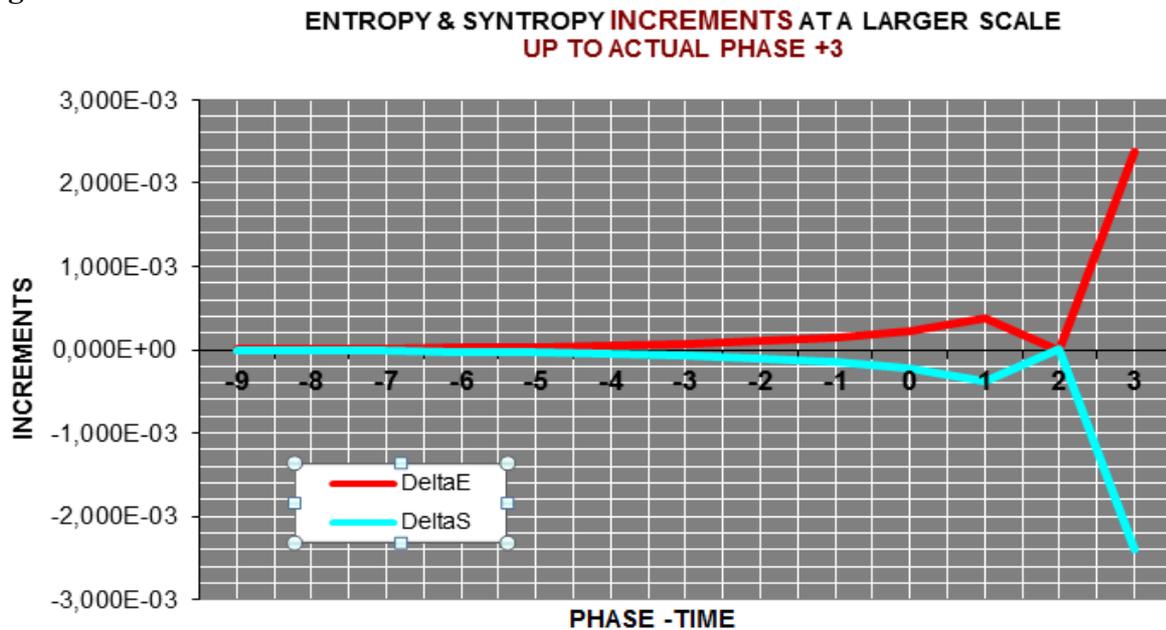
The sequence of *virtual phases* is mathematically unlimited. It can be stopped according to the decimal number approximation of the simulation figures that the simulation operator considers as substantially coincident with the figures of the *original standing state.*

   Note: At variance with the *actual* transition phase equations*,* all the equations concerning the *virtual phases* of the simulation, like the equations [4.25], [4.26]….[4.31] etc., are *not simultaneous equations*; moreover, all of them imply positive solutions, necessarily, for each unknown term of the sequence is individually determined through either the product or the sum of known terms, all of which are positive quantities. This may also be viewed as a way to represent the *certain* past of any evolution process versus the simulation of the *uncertain* phases of its future development.

   (The presentation of the *transformation cycle* by use of the interaction *probability* configurations is substantially identical to a presentation made by use of the interaction *flow* configurations, which – for the sake of concision – I do not deem necessary to explicit).

4.4 *Some logical implications of the simulated evolution*

   As seen, from the original standing state up to the *actual phase* of the simulation cycle that requires the *transformation*, the structure of the system remains the same as that of the original configuration. Therefore, it is licit to say that:

(1) The structure of any original configuration *is also the structure of* every *simulated transformation cycle that may start from the same original configuration*. Considering the equations of the kind [4.15] and [4.15a] as pertaining to transformation cycles, the relevant structure may also

be expressed by *the set* of the two square matrices of coefficients $\{A_{jk}\}$ and $\{B_{jk}\}$ defined as follows:

[4.32] $A_{jk} = Y_{j.}\varepsilon_{jk}$, $(\forall j, k)$

[4.33] $B_{jk} = X_{k.}\varepsilon_{jk}$, $(\forall j, k)$.

(2) Given the *cycle structure* (as defined above) together with *one* semi-base of any *transition phase* of any transformation cycle, the entire sequence of that particular transformation cycle is univocally determined, including the *original standing state* calculated as *asymptotic virtual configuration* of the cycle.

(3) Given the configurations of any two transition phases (*i*) and (*i*±1) *that share a common semi-base* (which works as the *transition vector* between the two configurations), the structure of the relevant transformation cycle is univocally determined by use of the equations [4.15] and [4.15a]; from which one obtains:

[4.34] $$\{A_{jk}\} = \left\{ \frac{P_{jk}^{(i)}}{Q_{k}^{(i)}} \right\};$$

[4.35] $$\{B_{jk}\} = \left\{ \frac{P_{jk}^{(i\pm 1)}}{P_{j}^{(i\pm 1)}} \right\},$$

depending on what semi-base forms the *transition vector*. Therefore, as per Point (2) above of this paragraph, all the remaining transition configurations of the transformation cycle are also univocally determined, plus (asymptotically) the configuration of the *virtual* original standing state. Then, from the latter, through the equations [3.30] and [3.33], it is possible to determine the relevant *intent matrix* $\{\mu_{jk}\}$ of the system. [31]

(4) If, in addition to that which is supposedly known as per *either* Point (2) *or* Point (3) above, the cycle's phase that *transforms* into a new standing state is also fixed, then the structure of *any next* transformation cycle is univocally determined too.

4.5 *Time, age and other parameters of the simulated evolution*

As already remarked in previous paragraphs, the time of the system's simulated evolution is beaten by the sequence of the transition phases of each cycle. The duration of each transition phase may reasonably be correlated with the time unit adopted to quantify the interaction flows of the system's original configuration (for example, flows detected in one hour or week or year, so that each transition phase may be thought of as the configuration's changes occurring in one hour or week or year etc., respectively); whereas there is no logical univocal way to establish the watch/calendar duration of any of the *standing states* that the simulation describes between

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[31] This property of the simulation can usefully be applied, for example, to the study of regions or areas affected by critical socio-economic conditions. A couple of subsequent surveys - *i.e.,* a baseline survey carried out before and a second one after any intervention aimed at coping with the critical issues - may provide data for both significant projections concerning the likely effectiveness of the measures taken and a better understanding of the possible initial causes of the present predicament.

subsequent transformation cycles. The *standing* states represent states of an *average unstable equilibrium* that may last one hour or one week or one year, respectively, or indefinitely longer. Thus, it is possible to assess the *presumable* watch/calendar time taken by each transformation cycle, though it would be only subjective guess establishing the expected duration of an evolution process that includes more than one transformation cycle.

Therefore, there is only a conventional definition of the *actual time* of an evolution process, which limits itself to reckoning the number of the *actual phases* of all the transformation cycles completed by the simulation, up to the particular *actual phase* under consideration.

There is also a conventional definition of the *age* of the system, which accounts for the intensity of the *labor* inherent in the *transition and transformation* processes, as these depend on the degree of efficacy of the interactions between the system's components.

Besides, there is to account for the "stress" affecting the system's evolution, which is also defined in an attempt to quantify, as a sort of common cost indicator, the overall amount of efforts, accidents and *troubles* that the system must face and overcome or undergo to proceed through any stage of its evolution

In addition, an "evolution level" is conventionally associated with the simulated evolution process, to quantify and express the effectiveness of the system's overall activity.

4.5.1 *The actual time (or evolution stage)*

The "**actual time**", or "**stage**", is a way to mark the position of the system in its evolution process. If $\tau_c^{\,p}$ is used to indicate the phase *p* of the transformation *cycle c*, then the *actual time* $\vartheta_c^{\,p}$ of the system at that phase is expressed by

$$[4.36] \qquad \vartheta_c^{\,p} = \tau_c^{\,p} + \sum_c \sum_{i=0}^{x_c} \tau_c^{\,i} \,,$$

in which $0 \leq i \leq x_c$ is the *variable* number of actual phases belonging to each particular transformation cycle *c*. For example, if $\vartheta_3^{\,2}$ is *the stage* of the system at the *phase 2* of *cycle 3*, then the *actual* past evolution time is in this case calculated as follows:

$$[4.37] \qquad \vartheta_3^{\,2} = \tau_3^{\,2} + \sum_{i=0}^{x_1} \tau_1^{\,i} + \sum_{i=0}^{x_2} \tau_2^{\,i} \,,$$

in which x_1 and x_2 represent the number of *actual* phases belonging to *cycle 1* and *cycle 2*, respectively.[32]

4.5.2 *The a**ge of the system***

The theoretical assumption by which the system's structure remains constant during the transition phases of the relevant cycle is only a way to escape the problem of representing the combined sequences of different and asynchronous minimal – albeit discrete in quantity – oscillations of the activators' expectations, while the system's configuration changes irreversibly.

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[32] Unfortunately, in this case, the general mathematical representation of the concept seems remarkably more complicated than the concept itself.

Such a fickle set of innumerable and *mutually out-phased* changes in the interaction intents is an extremely complicated process of individual re-adjustment of the activators' behaviour to the effects of their interactions. It may be thought of as a temporary *frantic oscillation* of the system's structure (represented by the *intent matrix*) around its original standing equilibrium (like, in a similarity, the structure of a building that trembles under the combined action of a vertical and wavelike earthquake); until the activators are compelled to *reset* and stabilize their expectations, with a view to keeping any possibility of interaction viable, *i.e.,* to avoid the system's collapse.

In itself, the *transition process* entails a temporary production of additional disorder (entropy), which – through the *transformation* of the structure – *may* be overcompensated by a re-organization of the system that makes its internal (*inevitable*) *dose of disorder* drop to a level significantly lower than the original one. In this connection, looking at the example represented by the data of **Figure 4.9a** and by the graph of **Figure 4.9b**, one can observe that the production of entropy (disorder) prevails during the whole transition of the system from its original standing state up to the *third actual phase* of the simulated cycle (*i.e.,* right up to the phase that is immediately preceding the *transformation)*.

The concept of "**age**" that is meaningful in this context relates to the amount of entropy produced throughout the system's evolution process. Therefore, this theory provides an assessment of the system's "age" through the ratio of the entropy E_c of the actual standing state to the entropy E of the original standing state, multiplied by the number c of completed transformation cycles (that is the number of undergone transformations), according to the following formula:

$$[4.37] \qquad a_c = a_0 + \frac{E_c}{E} c \;,$$

where a_0 is an arbitrary "initial" value assigned to the age of the original state; a_0 may either occasionally or conventionally be assumed as equal to 1. "Age" is here a *relative concept* that *adds age* to a pre-existing age however determined for the system. [33]

Thus, among possible different evolution processes with the same number c of transformation cycles, simulated either for the same system or for different systems of equal size, the highest ratio E_c/E identifies the evolution sequence affected by the most intense *aging process*.

With reference to the example exercise proposed in the previous paragraphs, the age of the system *after the first transformation* cycle is given by

$$[4.38] \qquad a_1 = a_0 + \frac{E_1}{E} \times 1 = 1 + \frac{1.669885}{1.7041659} = 1.979884 \;,$$

assuming $a_0 = 1$.

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[33] The "evolution parameters" introduced in this and in the following paragraphs are rather to be considered as conventional *indicators*, which are defined in view of practical applications of the theory to real problem cases. In the original version of this theory, the "parameters" were in part slightly different; furthermore, the simulation operators have full liberty to modify such para-meters according to particular needs of the study in progress.

4.5.3 *Development level*

During simulation exercises, it may be useful considering an indicator of the "degree of development" achieved by the system through the simulated evolution. For this purpose, an appropriate indicator, meant by symbol Λ_c, is the following one:

$$[4.39] \qquad \Lambda_c = \frac{S_c - S}{a_c},$$

in which S_c and S are the system's syntropy after c transformations and the syntropy of the original state, respectively.

Quantity Λ_c may be either positive, nil or negative. In the latter case, the simulation has entered the description of a *recession process* of the system. The simulation regards a *stagnation state* of the system if $\Lambda_c = 0$.

4.5.4 *Standing equilibrium, phase deformation and stress*

To proceed with the definition of additional phase parameters, it is worth going back to the analysis of the condition that defines the standing equilibrium state of the system. It has already been remarked that the equations pertaining to the equilibrium state are characterized by the presence of the *state factor h*, whose definition (refer to the equation [3.33] and subsequent ones) is given by

$$[4.40] \qquad h = \frac{1}{\sum\limits_{j,k} P_{jk}^{*}\varepsilon_{jk}} .$$

Remembering the equations [3.45], and bearing [3.1a] in mind [by which $P_{jk}^{*} = P_j Q_k$, $(\forall j, k)$], from [4.40] one obtains:

$$[4.41] \qquad \frac{1}{h} = \sum\limits_{j,k} P_j Q_k \varepsilon_{jk} = \sum\limits_{j} P_j \sum\limits_{k} Q_k \varepsilon_{jk} = \sum \left(P_j \frac{P_j}{Y_j} \right).$$

Then, (refer to [3.40]) considering that $h = \sum Y_j = \sum X_k$, the equation [4.41] becomes

$$[4.42a] \qquad \sum \frac{P_j^2}{Y_j} = \frac{1}{\sum Y_j} .$$

Analogously, one obtains also

$$[4.42b] \qquad \sum \frac{Q_k^2}{X_k} = \frac{1}{\sum X_k} .$$

These two equations *together* define the *standing equilibrium condition* of the system. As visible, this definition does not involve the interaction probability distribution, but only the system's base and the relevant structure potentials.

Albeit keeping the structure potentials $\{Y_i\}$ and $\{X_i\}$ constant during the transition cycles, each transition phase is characterized by *transition vectors* (which constitute *temporary transition bases*) that differ from the system's original base. Therefore, the condition expressed by [4.42a] and [4.42b] is no more verified. Instead, it is observed that

[4.43a]
$$\delta_P^{(i)} = \sum \frac{(P_j^{(i)})^2}{Y_j} - \frac{1}{\sum Y_j} \neq 0$$

[4.43b]
$$\delta_Q^{(i\pm1)} = \sum \frac{(Q_k^{(i\pm i)})^2}{X_k} - \frac{1}{\sum X_k} \neq 0 \,,$$

where, $\{P_j^{(i)}\}$ and $\{Q_k^{(i\pm1)}\}$ are the *transition semi-bases* of any two subsequent transition phases (i) and $(i\pm1)$, respectively. Quantities $\delta_P^{(i)}$ and $\delta_Q^{(i\pm1)}$ are dubbed "**phase deformation on P** " and "**phase deformation on Q** ", respectively. After division by h, the same quantities define the respective "**phase stress**" as follows:

[4.44]
$$\sigma_c^{(i)} = \frac{[\delta_P^{(i)} + \delta_Q^{(i-1)}]_c}{h_{(c-1)}} \,, \quad \text{or else} \quad \sigma_c^{(i)} = \frac{[\delta_Q^{(i)} + \delta_P^{(i-1)}]_c}{h_{(c-1)}} \,, \; (\forall i, c) \,,$$

depending on which transition vector starts the initial actual phase of the cycle considered. The index c refers to the particular evolution cycle regarded, while $h_{(c-1)}$ represents the *state factor* relevant to the system's standing state immediately preceding the transformation cycle c in progress. Then, by sum on the superscript (i) of the *phase stresses* [4.44], also the "**cycle stress**" remains defined for each particular transformation cycle c, as expressed by:

[4.45]
$$\chi_c = \sum_{i=1}^{x_c} \sigma_c^i \; ; \qquad\qquad (\forall c) \,.$$

In simulation exercises concerning the search for alternative possible solutions to analysis or planning problems, the *stress indicators* may help to identify the most convenient choices, considering that both *phase* and *cycle stress* are associated with the collective effort undertaken by the system's components to transit from any standing state to the subsequent one. There is to consider that the numerical values of the stress indicators may be either positive or negative: The relevant significance depends in particular on how such values contribute to the system's stability, to which they are obviously connected through the state factors h_{c-1} .

5. Possible uses of the theory – Defining the system's mutations

The contents of the preceding chapters are substantially summarized by the schematic example of simulation carried out in Chapter 4. That example exercise intends to show that the main aim of this theory is to provide any interested reader with a workable method for tackling issues that regard the behavior of "complex systems", especially in simulating their possible evolution processes. Many of such processes, particularly those that regard the activity of biological communities of a higher level of organization, do not allow observers and analysts to carry out *scientific* experiments and tests. It seems then necessary or convenient *shifting* observation and tests to the level of

reasoning, focusing on the logical consistency of the mental processes that utilize the observation data collected.

The ways in which the method can be applied entail the use of adequate, albeit not complicated, computer programs and relevant computerized work, which basically consists of numerical calculations, in consideration of the very large number of data that shall usually be collected and processed as to the behavior of complex evolving systems.

Indeed, any satisfactory simulation can be performed only through a large number of reiterated trials, each of which must be prepared through a series of preliminary checks and statistical adjustments on the primary data collected.

It seems also worth observing that the generality of the features that make a system recognizable is a significant aspect of the method here introduced; which suggests that the method is usable in a variety of possible study cases.

5.1 *Regional economic systems*

The origins of this theory are in the initial attempts to model the inter-relations between human settlements of a region, which involved – as it normally involves – difficult survey operations. Difficulties relate principally to the identification, selection and quantification of the relations that could make the simulation of any regional evolution significant.

As an acceptable simplification, in a number of applications of the theory to real cases, it was assumed that the flows of people and commodities from one place to another – as observed at any scale - constitute the most significant set of *interactions*, which – once detected and suitably quantified – can allow the observer/analyst to try useful descriptions and simulations of the regional activity and development process. The assumption is based on the observable fact that all other kinds of interrelations, such as those that occur via postal, telephone, Internet, mass-media communication, as well as through financial transactions, manifest their physical effects through displacement and resettlements of peoples and commodities. It is the system of detectable *motions* that alter with time the physical aspect as well as the *functions* of any studied region. In other terms, the interactions between different people and distinct settlements of a region consist in flows of *information*, which - through the displacements of people and commodities - transform into physical motions and landscape changes.

However, technical difficulties, particularly in establishing a homogeneous measurement system for the different kinds of flows observed (for example, transfers of people and transfers of commodities) lead analysts to deal often with different interactions by means of separate analyses and simulations, whose findings shall subsequently be *superimposed* or *combined* somehow in view of credible syntheses and conclusions. [34]

As far as economic systems are concerned, an interesting suggestion comes from that particular section of econometrics that addresses the interactions between the *activity sectors* of any regional economic system, with no involvement of the philosophical postulates (or axioms or universally shared assumptions) and categories proper to the thought of political economics.

The trouble with macro-economics and political economics in face of the real world is the same as the trouble that affects philosophical thought since ever: The unexpressed (that's why it seems impossible to eradicate it) axiom by which the "territory" of reality *must* coincide with the "maps"

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[34] For practical purposes, in most transportation studies engineers adopt a measurement criterion based on the so called "*passenger car units*" (*pcu*), which seems adequate to homogenize transported quantities of both people and commodities. (An example of *partial* application of this theory to a transportation study is in: www.mario-ludovico.com/pdf/traffic_flows.pdf).

of thought. As a matter of fact, the appeal of theoretical macro-economics and political economics is so charming that if one enters that world then finds it almost impossible to get rid of that peculiar way of thinking; like difficult is for people affected by schizophrenia or alcoholism to get rid of hallucinations. [35]

American economist Wassili Leontiev (1905-1999) moved his own theoretical thought from the charming cages of political economics to a sharp pragmatic way of thinking, in the attempt to suggest criteria for a more realistic analysis and – possibly – for a more effective control on the economic system of any country or region.

Leontief proposed a new method to make out any economic system and to probe its sensitivity to the changes expected in consequence of political decisions involving the economic activity of the relevant region. The method formulated by Leontiev did initiate that branch of econometrics known as *analysis of inter-sector economic relations*, more frequently and briefly mentioned also as *input-output analysis.*

The leading concept consists in subdividing, in a way that is as far as possible detailed, the whole regional economy into a system of different inter-related sectors of activity, and in quantifying the transactions occurring between such sectors during any given time unit, for instance during one year.

The method sticks to the statistical *measurement* of the interaction flows between sectors of a given economic system, with no other "prejudice" than the criterion adopted for identifying the various inter-related sectors. To note that such a kind of statistics is nowadays available in every developed country and in several developing ones.

From the methodological point of view, however, the choice of the criterion for identifying the "economic sectors" is inevitably sufficient to undermine the *full* objectivity of the analysis, as it also happens in identifying and recognizing any system. There is to consider that each "economic sector" consists in the *thought* aggregation of different activities, some of which – because of the respective typology – might be included in one sector instead of another, according to the analyst's criteria.

The production (*output*) of each sector is strictly depending on the portions of its production purchased (*i.e.,* demanded as production *input*) by the other economic sectors.

The only hypothesis, which constitutes also the crucial technical limit of the method, is that the *input* of each sector is directly proportional to the respective *output*. In principle, concerning the production system, the hypothesis is hardly questionable: Everybody would agree, for instance, on that the amounts of coal, mineral materials, labor, energy, capital money, transport costs, etc., are directly proportional to the amount of steel produced; and analogously for other sectors.

By this criterion, Leontiev could construct a numerical table (matrix) of *inter-industrial relations*, which basically consists of proportionality coefficients, usually mentioned as "*production technical coefficients*", to be assumed as constant quantities.

The basic idea is simple. Example: to produce *A* tons of steel, it is necessary to buy *K* kilograms of coal, *I* kilograms of iron mineral, *W* watts of energy, *M* hours of manpower, *F* dollars of financial services, *T* dollars of transport, etc. Leontief's method assumes that the numerical ratios defined by *K/A, I/A, W/A, F/A, T/A,* etc., keep constant with time, quantities *K, I, W, F, T,* etc., being in turn partial products of other activity sectors of the same economic system. The assumption can be summarised saying that the purchase of these quantities is in a direct proportion to the quantity of the product *A* regarded. In principle, it's quite a reasonable assumption.

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[35] Unfortunately, it seems that the trouble is also with basic physics and cosmology, at least since half a century. A worldwide overcrowded club of highly skilled mathematicians-conjurors have involved human societies in huge and foolish expenditures to *create* – instead of *finding* – the objects of their own mathematical deliriums.

Therefore, analogous obvious considerations apply to any production activity of the system**;** so that a set of simple simultaneous (inter-related) linear equations can be written to describe the system of relationships by which each activity is tied to all the other ones. In this way, it is possible to calculate, for example, the extent to which the product of the whole economic system depends on alterations in the production of any individual activity sector.

[One only example equation should be sufficient to make the criterion clear. Consider an economic system formed by *N* different economic sectors, indicated with "*1*", "*2*", "*3*", … , "*n*". Assume that the amounts of the yearly sector productions (*outputs*) are symbolised by $x_1, x_2, x_3, … , x_n$**,** respectively.

According to Leontief, it is possible to establish **fixed ratios** { a_{ij} } – dubbed "**technical coefficients**" – between each sale q_{ij} from any economic sector "*i*" of the system and the production of any buying sector "*j*". Thus, the *technical coefficients* relevant to the sales of production x_i are defined as follows:

$$a_{i1} = q_{i1}/x_1; \quad a_{i2} = q_{i2}/x_2; \quad … \quad ; \quad a_{in} = q_{in}/x_n ,$$

which simply means that **each sale** $q_{i1}, q_{i2}, …, q_{in}$ from Sector *i* is directly proportional to each production $x_1, x_2, …, x_n$ of the respective buying sector.

The sale from Sector "*i*" to each other sector is clearly just one portion of what "*i*" produces, *i.e.*, a portion of its overall production. x_i . Therefore, production x_i can be expressed as the sum of the *demand* from all the other sectors, each *demand* being directly proportional to the production of the respective sector. Mathematically, this means that production x_i can be expressed as a linear combination of the other sectors' productions, each of which is multiplied by the pertaining *technical coefficient* (needless to say, every *technical coefficient* is a number less than 1).

Adopting monetary units to homogenize the measurement of whatever product, the situation is then summarised by the following simple equation**:**

[5.1] $x_i = a_{i1} x_1 + a_{i2} x_2 + … + a_{i2} x_n .$

It's now clear how the total *output* x_i of any Sector *i* may vary in consequence of changes in the productions x_1 and/or x_2 , and/or x_3 , etc., of other sectors, for each addendum $a_{ij}x_j$ is the demand of *j* for its production factor sold by *i.*

 Identical reasoning is applied to every other sector, to write the set of simultaneous equations that describe in precise quantitative terms the interdependence between all the activity sectors of any economic system. **]**

One practical problem in applying the method arises because each of the identified sectors does not consist of a single type of production plant, but the sector – due to an inevitable need for simplification – aggregates the outputs of several different activities, which are *akin* but not *identical* to each other: So that the inter-sector transactions cannot be measured in homogeneous product units (*e.g.*, in tons, or cubic meters, etc.) but only as transaction flows expressed in monetary units.

Additional practical difficulties intervene when the analysis aims at long term predictions, which cannot necessarily account for the immanent *disturbing* role of technological innovation and unforeseeable changes in the price/cost of some inputs or in unpredictable market or governmental policies.

Notwithstanding the inherent practical difficulties, Leontiev's conceptual approach to the economic macro-analysis is revolutionary, in that it does not break down the study system into selected conventional economic categories (labor, capital, investment, marginal utility, elasticity of demand, offer, market equilibrium and/or transparency etc.)**.** Instead, the analysis limits itself to identify and account for transactions between different *activities*, intrinsically and objectively measurable irrespective of their nature and of any cause or end that determines them. The methodological scheme, in other words, may be applied to any society and economic system, provided that the basic assumption is verified, *i.e.*, that an acceptable degree of linear inter-

dependency between the different activities exists. In itself, Leontiev's inter-sector analysis has no reference to any particular school of economic thought.

Beyond all possible criticism, it is an important attempt to free macro-economics from abstract philosophical speculation, with a view to keeping the observation of a complex system within a *least-biased* conceptual reference frame.

As known, after the original scheme proposed by Leontiev, the method has undergone a remarkable number of improvements and adjustments, and the *input-output* inter-sector analysis has been adopted by several governments for managing national accounts. It is a fact that the method, despite the approximations associated with the hypothesis of linear dependency between the system's activities, provides analysts with a useful *calculation* instrument to get credible short term indications about the expected impact on the whole system caused by possible alterations in the activity of one or more of its sectors. No other model can provide the analysts with a credible *objective* indication of what impact, for instance, on clothing industry could be expected from an increased investment in automobile industry, or what impact on fishery production could be connected to a decrease in the family savings.

Actually, the method constitutes the first *usable* instrument of *complex systems analysis*. The observation and measurement of interactions between human activities, along with the identification of the functional nature of the relationships, accounts for *all* that which motivates and determines the behavior of the members of a self-organized human society, including the *chaotic* set of individual intentions, prejudices, errors and superstitions. *All this* is completely, as well as indistin-guishably, expressed by the intensity of the measurable transactions.

5.1.1 *A further step*

The methodological jump made by Leontiev in addressing macro-economic issues is an encouraging suggestion to go further along the conceptual path he has indicated.

Leontiev's inter-sector analysis, as already remarked, is affected by at least one ill-working *functional* hypothesis, the one regarding the "technical coefficients" of direct proportionality between inputs and relevant outputs. The analytical need for the aggregations of various *akin* different activities makes the direct proportionality between *input* and *output* not only questionable, but systematically unstable with time, mainly – but not only – because of frequent alterations in the prices of the production factors along with unforeseen productivity changes in some of the activities considered. The method would be quite adequate, especially as for short run projections, if the "technical coefficients" would be constant quantities. Unfortunately, experience has widely shown that it is not so. This fact has actually implied a complicated and endless work of formal adjustments of the method together with a continuous updating activity concerning the set of values forming the matrix of technical coefficients.

Moreover, there is to consider that the simultaneous equations, of which equation [5.1] above is a sample, does not usually escape the mathematical problem of providing positive solutions only. A possible remedy to this particular problem could be dealing with equation unknowns that represent production *increments* instead of total production amounts (*i.e.,* $\{\Delta x_i\}$ instead of $\{x_i\}$).[36], but such a remedy implies additional statistical work for pointing out the sector production increments and checking to what extent the relevant *technical coefficient*s reflect the proportionality expected.

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[36] At variance with the total production of each sector, sector production *increments* $\{\Delta x_i\}$ may be either positive or negative or null.

That is why the management of national and regional accounts by means of the input-output analysis suggested by Leontiev has in most cases become relatively burdensome and costly, with results often unreliable though.

According to Leontief's initial approach to the analysis of an economic system, the inter-sector relationships, as per time unit, show the following configuration:

$$
[5.2] \quad
\begin{array}{c|cccc|c|c}
\textbf{?} & T_{01} & T_{02} & \dots & T_{0n} & T_{0F} & Imp \\
T_{10} & T_{11} & T_{12} & \dots & T_{1n} & T_{1F} & T_1 \\
T_{20} & T_{21} & T_{22} & \dots & T_{2n} & T_{2F} & T_2 \\
\dots & \dots & \dots & \dots & \dots & \dots & \dots \\
T_{n0} & T_{n1} & T_{n2} & \dots & T_{nn} & T_{nF} & T_n \\
\hline
Exp & D_1 & D_2 & \dots & D_n & D_F & T
\end{array}
$$

In this transaction configuration, T_{jk} – as usual – represents the transfer of goods or services from any Sector j to any Sector k, index "0" relates to the production sectors of "the rest of the world" (*i.e.,* the "external sector"), "*Imp*" represents the total amount of imported products, "*Exp*" represents the total amount of exported products, "D_j" represents the total demand of any Sector j for production factors, "T_{jF} " represents the amount of production sold by any Sector j to the so-called "Final Sector", which is the "Consumption Sector". $T = \Sigma T_j$ represents the overall production of the economic system. The elements "T_{jj}" in the diagonal of the inter-sector transaction matrix (the *self-interactions* of the economic system) represent the transactions occurred *within each sector* between distinct *akin* production centers.

One of the economic sectors in the scheme was reserved to the *labour input*, which – though being a major production factor – was not the destination sector of any product, since "one side" of the *labour sector* was included as a *product consumer* also in the so-called *final sector*. A subsequent more compact configuration of Leontiev's input-output scheme removed the *final sector* and introduced the "Household Sector" as labour provider and product consumer, thus *squaring* the input-output matrix. Obviously, the "technical coefficients" that state the direct proportionality between the households' earnings (*output,* in terms of *sold* manpower) and the households' expenditure (*input,* as purchase of goods and services)[37] is only an additional hypothesis, which, in the many contexts of practical applications of the method, is not unreasonable.

The question mark "*?* " in the position of the first element of matrix [5.2] represents the *unknown* self-interaction of the "external sector". In Leontiev's scheme, sector self-interactions are usually important enough not to be neglected. If, for example, the aggregation of activities entails – for simplification needs – the inclusion of the industrial factories that produce steel, metal tools, machinery, automobiles and the like, in one and the same "Metal & Mechanical Sector", it is clear that exchanges of products between different factories, within the same sector, are inevitable. Only at a much lower level of aggregation of the economic activities (for instance, with reference to the preceding example, keeping steel-mill factories, rolling mill industries, tool factories, motor-vehicle industry etc., as *economic sectors* distinct from each other) it could practically be acceptable assuming that the sector self-interactions are negligible with respect to the transactions between each of such sectors and all the remaining sectors of the economic system.

However low the aggregation level may be, the self-interaction " *?* " of the external sector ("the rest of the world") can in no case be considered as negligible. This means that Leontiev's interaction system can never be *closed*: There is no acceptable way for completing the squared scheme with a justifiable figure in replacement of the question mark; no logical instrument is available to solve the problem by connected econometrical means.

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[37] Any form of savings concerning households may be considered as *self-interaction* of the household sector.

This fact, amongst other different cases of interactions between human activities, has suggested the leading criteria to be adopted for formulating the theory expounded in this paper.

By exclusion of any hypothesis concerning either technical or market *causes* that determine the inter-sector transactions, and accounting only for the statistics of the interaction flows assessed in terms of monetary amounts exchanged between the sectors, the *revised* input-output analysis of any economic system allows, *through a logical procedure*, the determination of the most probable *implicit flow*, by which the representation of the economic system can both be *closed* and *probed* by simulation of *possible* evolution processes.

Perhaps, it is not necessary to underline the utility of this new instrument to *probe* the likely effectiveness of *possible* socio-economic policies. Just concerning economic systems, it is also worth noting that the new method is not troubled with problems relative to the aggregation of more or less *akin* activities: What matters is only that the different sectors are thought of as playing clearly differentiated roles within the economic system considered.

### 5.2 *The case of a system evolution at constant stability*

In the last paragraphs of Chapter 2, the evolution of any system has been graphically suggested as describing arcs of ellipses, whose major axes represent the entropy potentials of the evolving system.

The *shape* of an evolution ellipse remains in general unchanged until base entropy and base syntropy remain unchanged too; which also means, obviously, until the system's *stability* represented by the base syntropy S^* remains constant.

However, as usually shown by the transition phases of a simulated evolution, the hypothesized (or observed) changes in the system's base entail redistribution of the base probabilities in a way that implies also changes in the stability of the evolving system, while the major axis of the evolution ellipse remains unchanged with the number of the system's components, these determining the system's potential.

In general, once fixed the size of its major axis, the shape of the ellipse changes with the shifting of its focuses, as their position determines the size of the minor axis. The shift of the focuses takes place along the major axis according to the extent of the *base* syntropy (or of the *base* entropy), which in turn depends of the distribution of the *base* probabilities.

Figure 5.1
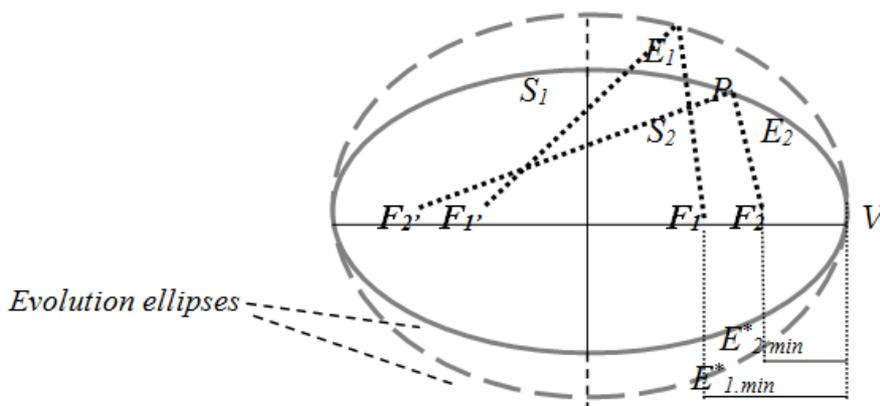
This figure recalls the features of the mentioned evolution ellipses
The common major axis represents the system's entropy potential H=2lnN

Nevertheless, it is possible to think of evolution processes that develop at constant stability, thus keeping the system's evolution path on the stable perimeter of the relevant ellipse. Remember that innumerable different configurations – regarding either the same system or different systems – may share a common *base* (refer to Paragraph 2.9), whereas different conjugated *syntropy* and *entropy* levels (*i.e., S* and *E*) are associated with each different interaction distribution.

In order for any evolving system to keep its stability constant, it is sufficient that the ratio of the overall activity of each component (*i.e.,* input+output) to the system's total activity remains constant.

Actually, stability in evolution may in general occur if changes *in the base probability distribution* are associated with a mutual compensation in the values of the addends of the sum that defines the formula for the *base entropy*; in this connection it is worth remembering the definition

[2.39] $E^* = \Sigma(P_i \ln P_i + Q_i \ln Q_i) = 2\ln T - (1/T)\Sigma(O_i \ln O_i + D_i \ln D_i)$

given for *base entropy* and the definition [2.42], *i.e.*, $S^* = 2\ln N - E^*$, given for *base syntropy* , which measures the system's stability.

Constant or quasi-constant stability intervenes in the simulated evolution process also when the system's syntropy approaches closely **-** or oscillates in a close proximity of **-** its possible maximum. It is worth bearing in mind that any system cannot exceed a specific maximum syntropy, for it cannot compress the amount of its intrinsic disorder (entropy) below specific levels. (Referring to **Figure 5.2** above, when position *P* joins point *V, System 2* achieves its maximum level of *syntropy*, while its *base* entropy $E^*_{2.min}$ - which at point *V* coincides with the system's entropy E_2 - cannot be reduced further).

Every system brings in itself a *physiological* amount of entropy (disorder) that is inherent in the recognition-definition of system adopted by this theory. The interaction distribution, in which any described system consists, is *by definition* affected by an irreducible amount of *uncertainty*, which depends on the lack of information or knowledge that *hinders* a complete perception and description of the *causes* that determine the *end* and the intensity of each interaction.

Therefore, especially in systems that have achieved a relative level of *maturity*, the degree of internal organization attains a relevant optimum, beyond which a further increase in the system's syntropy is unlikely or impossible: Unless a *mutation* intervenes to change the entropy potential of the system, the achieved *standing state* tends to become a *stable one*.

For example, referring again to economic systems, persistent stability in their evolution may in a number of cases be associated with a gradual replacement of previous activity sectors by emerging activities that adopt new technologies, though producing commodities and services of the same kind as those replaced, often shifting the respective production amounts from one sector to another one, for example from craftsman-ship to industry or from agriculture to industry, or from the mining sector to the energy sector, etc. In this way and in particular cases, the *base probability distribution* concerning percentages of sector aggregate products doesn't modify the *base* syntropy/entropy significantly, with the consequence of securing a longer stability for the system.[38]

Besides, *stability* may also characterize *involution processes*, in which systems achieve a state of rather stable equilibrium after moving towards states of increasing entropy. In the real world, it is a process that occurs when the components of a system tend to simplify and isolate the respective activities, and the interactions become less and less structured by a network of interdependent *intents*. The randomness of the interconnections tends to increase progressively, because the kind of

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[38] An illustration of this kind of replacement process is given in the attachment herewith (*Syntropy: Definition and Use*, reviewed excerpts).

activity of each component tends to differentiate very little from the activities of the other components of the system. Consider, as an extreme example, a hypothetical set of small countries - in a particular geographic region – where the principal activities are quite similar in all of them and substantially consisting of would-be self-sufficient agriculture and animal husbandry. The economic interactions between such countries are often non-systematic, consisting of occasional exchanges of agricultural and/or local craftsmanship products. [39]

5.3 *Mutation in a system*

Any system shall be understood as a set of *different* components, which interact with each other because of the need to *integrate different complementary functions*. Thus, if it is a human system, each individual plays a different role in self-connecting with other individuals, like in the example suggested in Paragraph 2.6. Or else, if it is an economic system, the relevant *sectors* play different economic roles: The number of sectors does not increase by adding one or two new metal-mill factories to the existing ones in the relevant sector, but only by addition of a quite novel economic sector, such as – for instance – a motor-car/automobile industry, or electronic industry where such sets of activities are absent.

Symmetrically, the number of components of any system does not decrease because of the diminution in the overall activity of one or more components (*e.g.*, if a couple of factories of those belonging to the textile sector cease their activity), but because of the disappearance of one or more components and relevant functions; as it happens, for example, when a basic extraction industry (such as coal or copper or iron mining activity) disappears from a regional economic system or when the section of wind instruments is definitively removed from a symphony orchestra. That is to mean that *the number of the system's components* does not consist in the number of the *akin* activities carried out by each *component*, but only on the *number of different functions* that characterize the overall activity of the system. Therefore, *size N* of any system means "system of *N* components each fulfilling one of *N* different interrelated functions that pertain to the system considered".

The *emergence* or the *disappearance* of one or more new *components* is referred to as a "mutation" of the system under study.

The first effect of a *mutation* is a change in the system's entropy potential: *New* additional components imply an *increase* in the entropy potential, whereas the *disappearance* of components implies a decrease in the entropy potential of the system, as it is obvious as per the definition itself of "entropy potential" ($H = 2\ln N$).

Mutations in the evolution of any system may be expected, but they are in most cases unpredictable. If, in representing any system evolution, a particular kind of *mutation* is expected, the effects of the emerging event can be quantified only if the distribution and intensity of its interactions with the other pre-existing components of the system are known. New appropriate surveys on the new situation of the *real study object* are indispensable to undertake useful simulations. Consider that the introduction of every new component in the system means the

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[39] It is a situation that characterized a number of ex-colonies, especially in Africa, during a period that followed the end of the de-colonization process. Embryonic industrial sectors and updated technological activities established by the colonial regimes were dismantled or abandoned, thus *mutating* the local economies, and leaving only a basic infrastructure network together with "remains" of administration organisms that were not existing before the colonization era. (As early references, see for example H. Isnard, *Géographie de la decolonisation,* Presses Univ. de France, Paris 1971, or P. Bohannan & G. Dalton editors, *Markets in Africa*, Northwestern Univ. Press, Evanston 1962).

addition of a *new function* to the pre-existing ones. All the previous intents and interactions are modified together with the system's structure.

In some cases, mutation can be expected when the evolution brings the system to a relative *saturation* of either *syntropy* or *entropy*, otherwise the alternative is for the system to lay in a permanent stationary state; which is obviously possible though normally unlikely. Therefore, "mutations" may entail either a *progression* of the system towards a higher level of complexity (when the number of the components increases) or a *regression* (when one or more components disappear from the system).

Alternate series of *progression* and *regression* are not only possible but also observed, especially concerning human societies or economic systems.

Moreover, cases of *replacement* are also possible, when the emergence of a new component is "compensated" by the disappearance of another component. In such cases, the system's entropy potential does not change.

H
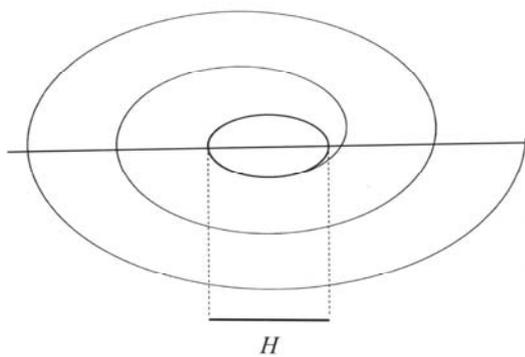
Considering that *mutations* are basically characterized by changes in the system's entropy potential *H, a spiral* like that shown by. Figure 5.2 beside may – schematically and in general – represent the path of any evolving and *mutating* system, for which *H* might represent the entropy potential of either the *initial* or *the final state* of its evolution. The spiral develops around and along its major axis (the *mutation axis)*, which contains all the *major axes* of the evolution ellipses in the system's story.

5.4 *Some quickly derived models*

In analyzing, for example, the human settlements of any study region, the interaction systems between population centres may usefully be represented by a set of so-called "gravitational models", which are based on the observed determinant role that the distances, between settlements of various sizes, exert on the intensity of the settlements' mutual influence. According to geographers who initiated regional science,[40] it is the distribution itself of human settlements over any territory to favor the formation of certain cities instead of others and to determine the topographical pattern of the respective hierarchical importance within the study region. Through patterns of that kind it is possible to recognize the existence of *regional systems*.

According to primitive models, the *link strength L_{jk}* between two population centers C_j and C_k is quantitatively expressed by equations of the following type

[5.3]
$$L_{jk} = g \frac{I_j I_k}{d_{jk}^2} = L_{kj} ,$$

in which I_j and I_k represent the number of inhabitants in C_j and C_k, respectively, d_{jk} is the distance between the two centres and g is a proportionality coefficient empirically determined, case by case.

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[40] Amongst the major forerunners, J. von Thünen, *Der isolierte Staat in Beziehung auf Landwirtshaft und Nationalökonomie*, Perthes, Hamburg 1826 (reprinted by Fisher, Jena 1921); W. Christaller, *Die zentralen Orte in Süddeutschland*, Fisher, Jena 1933; A. Lösch**,** *Die räumliche Ordnung der Wirtschafts*, Fisher, Jena 1940; many followed later.

The adjective "gravitational" was attached to models of that kind because of the formal similarity between the mathematical shape of equation [5.3] and the universal gravitation equation formulated by Newton to express "the attraction strength" between two massive bodies.[41]

Subsequently, "gravitational" equations adopted in regional science were variously reshaped, also in view of different particular uses aimed at specifying the nature of the *link*; as in the case of traffic studies, for example, where the "importance" of the traffic flows from any C_j to any C_k is not necessarily symmetrical with the flows that go in the opposite direction.

Notwithstanding the many changes intervened, gravitational models are still largely used in urban and regional studies because of their simplicity and *acceptable* efficacy in providing significant indications in practical planning activities.

If quantities I_j and I_k in the "gravitational" equations [5.3] are replaced, for instance, by quantities O_j and D_k, respectively, to indicate the actual or potential interaction origins and destinations, whatever the respective nature, in conjunction with the substitution of the distance d with any function of variables fit for quantifying the *friction of the distance* between the two centers, then the theory expounded in this paper presents its own "gravitational" proposal with equations like the equations [3.36] and the connected ones, from [3.36] to [3.49].

Equations [3.36] state that, given any system of N interacting centres, the interaction between any couple of centres is expressed by

$$[5.4] \qquad\qquad T_{jk} = h \frac{O_j D_k}{T} e^{\lambda u_{jk}} \qquad\qquad (\forall j, k),$$

in which – allowing for the definitions [2.15] and [2.16] introduced in Chapter 2 – the *friction of distance*, in economic terms, is expressed by

$$[5.5] \qquad\qquad \lambda u_{jk} = \lambda(b_{jk} - c_{jk}) \qquad\qquad (\forall j, k),$$

b_{jk} being the average "reward" obtained by the interaction flow from j to k, and c_{jk} is the cost of the same interaction, which obviously includes the cost to cover the distance from the two activity centres.

In the equations [5.4], as usual, $T = \Sigma O_i = \Sigma D_k$, while "gravitational" coefficient $h = e^{-S^*} = {} = \exp\{2\ln(T/N) - (1/T)\Sigma(O_i\ln O_i + D_i\ln D_i)\}$ remains constant with the system's base $\{O_i\}$ & $\{D_i\}$, and λ is also a constant coefficient, which depends on the measurement system used to quantify the interaction flows.

Simplifications are obviously possible, especially as to the quantity b_{jk}, which – being an *average quantity* – may in some particular cases be considered as a *relatively* constant "destination reward" with respect to all the interaction origins. In this way, the equations recalled by [5.4] become similar to the "gravitational" models that are more frequently used in regional science, in that the interactions are basically made depend on the cost associated with the distance. Thus, the simplified equations [5.4] become "gravitational" in the following form:

$$[5.7] \qquad\qquad T_{jk} = g \frac{B_k O_j D_k}{e^{\lambda c_{jk}}} \, , \qquad\qquad (\forall j, k),$$

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[41] On this occasion too, it seems worth reminding the reader that Newton himself **did not** consider his gravitation equation as an expression of attraction force *inherent in* masses.

where $g = (h / T)$ is the system's constant, and $B_k = \lambda b_k$ indicates that the *average benefit* B_k , which "rewards" every interaction unit bound for destination k and does not depend on the source of the interaction.

Upon field-surveys carried out on interaction systems, a reverse use of equations [5.7] could provide analysts with significant indications about the *economic distances* $\{\delta_{jk}\}$ between interacting centers, according to the following obvious relations:

[5.8]
$$\delta_{jk} = \lambda c_{jk} = \ln\left(\frac{gB_k O_j D_K}{T_{jk}}\right), \qquad (\forall\, j,\, k),$$

by which, positing B_k.=1 (or any other appropriate figure instead), it could be possible to assess the *relative* economic distances estimated from the *k's* standing point. Consider this, for instance, as one supplementary "tool" for the choice of convenient locations of new commercial centers or administrative structures in any given urban/regional area.

Other possible simplifications or uses of models derived from the theory should be allowed for in consideration of the canonical and semi-canonical equations [3.42] to [3.49], thanks to the system's structure potentials $\{X_i\}$ and $\{Y_i\}$ as per the respective definitions [3.41].

6. A few conclusions and philosophical comments

6.1 *Theory as method*

It is not a scientific theory. *The contents of the foregoing chapters constitute a methodological proposal only.* It is not a theory that purports to explain what a complex evolving system is. The text intends just to suggest an operational way to represent sets of events that an observer can recognize as being both in a "complex mutual inter-connection" and "evolving", according to his own mental and linguistic apparatus, discernment capacity and interested attention. In the awareness that all that which he learns by experience remains – intrinsically – only a mental representation of his experience; he certainly apprehends *neither* "the thing in itself", *nor* the *objectively true* features and *causes* of the *real* source of his experience.

Systems are subjectively viewed as physical flows of interactions between *activators* that are supposed to generate interactions intentionally, *i.e.,* with the purpose of achieving goals. I deem it is a basic and quite general assumption that may be shared by the majority of the observers who try to describe the activity of any self organized system. Therefore, the theory associates a different *average intent* with every different interaction observed, though it considers the observer as aware of the fact that "average intent" is only a statistical expression for a quantity that is impossible to seize in its true nature, and whose existence is a theoretical fiction used to represent a *guess* about one particular aspect of the reality under observation.

In this connection, not only are such *intents* defined by the theory as depending *only* on the *detectable* interactions they motivate, but each *intent* is also thought of as inherently affected by *precariousness*. This aspect is discussed in detail in the original Italian text of the theory,[42] where – besides the definition of *intent* given by the equation [3.50] in page 61 of preceding Chapter 3 – another (equivalent) definition of this particular quantity is provided also by the following equation

[6.1]
$$\mu_{jk} = H - \xi_{jk}, \qquad\qquad (\forall\, j,\, k),$$

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[42] See ***L'evoluzione sintropica…*** , *op. cit.,* pp. 118-122

where, as usual, $H = 2\ln N$ is the system's *entropy potential* and ξ_{jk} represents the intent's inherent *precariousness* as expressed by the following equations:

$$[6.2] \qquad \xi_{jk} = E^{*} - \ln\left(\frac{P_{jk}}{P_{jk}^{*}}\right) = E^{*} - \ln\left(\frac{T_{jk}}{T_{jk}^{*}}\right) \qquad (\forall\, j,\, k).$$

in which the involvement of the base entropy E^{*} is given evidence. Moreover, the interaction flows $\{T_{jk}\}$, $\{T_{jk}^{*}\}$, together with the respective probabilities $\{P_{jk}\}$ and $\{P_{jk}^{*}\}$, are also statistical assessments made by the analyst upon the findings of on-the-field surveys, which bear in themselves an inevitable dose of uncertainty associated both with the execution of the surveys and with the interpretation of the data collected.

Consider again that such a methodological proposal does not concern *every recognizable* complex evolving system, but *only* those systems that one can represent as consisting of *interaction flows* between couples of interaction origins and destinations, whatever their nature. Indeed, one salient aspect of this way to represent systems is that the analysis – at variance with other theories about complex systems – focuses neither on the "agents" (the *acting components* or "activators") of the system nor on the physical web (or infrastructure network) that connects each component to the other ones. The attention of the analysis is entirely focused on the *nature* and *intensity* of the *interaction flows* by which the system's components are interconnected. Furthermore, such interaction flows are *elements* that pertain to the theory *only if* they are practically measurable.

Equations [3.60] and [6.1]-[6.2] show how all the needed information concerning the identified system can be expressed through functions of the interaction flows.

From a practical point of view, as already remarked, all this means that the significant amount of information concerning the state of the system can be obtained – according to the theory – through any appropriate collection, interpretation and processing of the data that quantify the flows of interaction.

However, and this is the fundamental methodological statement, all the information obtained from the theoretical analysis depends *strictly* on *how* the system is recognized and described. The simulation exercises suggested by the theory do not provide any *true picture* of the reality to which the analysis refers, but *only* the logical implications of *a mental representation* of it.

Yet, the same methodological proposal makes sense *only* for those study cases that do not allow observers and analysts to carry out experimental activities in which the observed events can be replicated and controlled at will. In all the cases in which experimentation is both replicable and effective, any relative theory works usefully if it keeps in a dialectic relationsip with experience. This means – in those cases – that mental representations of experience can be subjected to practical tests, whose results can either corroborate or belie the related theoretical statements and predictions.

Instead, when no significant experimental test can be carried out, theoretical works must only rely on the soundness of their observational basis (*i.e.,* on the reliability of the data collected) and on the logical consistency by which the data are processed. Statements from such theories can only consist of *reasonable* illations and *probability assessments* of event occurrence.

The preceding Chapter 5 has lingered on the example of regional economic systems, for these systems are a paradigmatic example of the "systems" meant by the theory expounded in this paper. Consider that the *components* (the "activators") of regional economic systems, which are referred to as "sectors", are not individually and obectively identified and localized in space by the "observer", since each "sector" consists of various *never seen* akin activities spread over large territories. In

fact, each "sector" is a mental representation of a *group/class of activities*, which interacts with other goups/classes of activities, *irrespective of their locations and of the connection infrastructure used*, whereas the interactions between different sectors, and within each sector, are actually detectable, measurable in terms of monetary transactions and formally recorded by statistics.

6.1.1 *Simulation of the system's evolution*

The most important equations provided by the method are those which enable the analyst to simulate the system's evolution process.

Conventionally, this kind of simulation considers any flow distribution obtained from surveys (or other observation operations) as the representation of an *original equilibrium state* ("standing state") of the system under study. This original equilibrium state is intrinsically unstable, and the relevant observed configuration shall be taken as an *average configuration* about which the system fluctuates precariously.

"Intrinsic instability" means that reversible fluctuations in the flow distribution within the original configuration are inherent in complex systems, though some *irreversible* alterations will sooner or later appear to modify the system's activity irreversibly: *Any minimal **irreversible** alteration in the fluctuating equilibrium of the flow distribution (or in the relevant probability distribution)*, **which modifies also the base entropy of the system**, generates a corresponding particular *initial* phase of an irreversible "transformation cycle".

Summarizing, the evolution process is described by a sequence of "transformation cycles" that start from an *original standing state* of *apparent equilibrium.* Each cycle develops through discontinuous "transition phases", which are changes in the system's state, each phase being described by a different distribution of the interaction flows.

In every phase of a transformation cycle, the condition of the system is expressed by a set of parameters (*state* or *phase parameters*), amongst which *entropy, syntropy, stress* and/or *latent stability* are the most significant ones.

Any transformation cycle is identified by an "initial phase", which is determined by any observed irreversible change – however small – in the original flow configuration, *if* the irreversible change *modifies also* the system's *base entropy E^** or *syntropy S^**, the latter representing the system's *stability.* (Note that the system's *base entropy and syntropy* may remain unmodified with an indefinite number of particular changes in the system's flow configuration).

Therefore, the *initial transition phase* of an evolution process is not the *original state.* Instead, the *initial* transition phase is viewed as *one particular episode* of a *transformation process* in progress, which then continues through a sequence of *actual transition phases,* each representing a period of the system's simulated future. Thus, the *initial phase* is in turn supposed to be connected with the *original state* through a sequence of *virtual* transition phases that represent the virtual (not observed) "past story" of the *initial phase.*

It is worth remembering that a sequence of *actual phases* describes **one** transformation cycle, which may – or may not – be followed by further transformation cycles.

During each cycle, the continuity in the identity of the studied system rests on two sets of quantities, dubbed "*structure potentials*" and represented by matrices $\{X_k\}$ and $\{Y_j\}$**.** The value of each *structure potential* ranges between 0 and 1. The structure potentials remain constant with the system's structure $\{\mu_{jk}\}$ (the "intents") during the transition phases of each cycle, until they change *by transformation* together with $\{\mu_{jk}\}$ at the conclusion of the cycle. If no "transformation" occurs, the theory assumes that the system disbands at the end of the cycle.

Simulations are possible only if a complete set of "original" interaction flows is given. Amongst the system's components, there is always to account for the *external component* ("the rest of the universe"), this being inevitably connected and interacting with any kind of evolving system one may observe and describe. The interactions proper to the systems meant by the method include also the *self-interactions*, *i.e.,* every interaction internal to each activator of the system. However, no survey can detect and measure the self-interaction of the "external component", whereas interactions between the *external component* and the other components of the system are in general detectable and measurable. Nonetheless, the theory expounded in this paper indicates a logical procedure by which the *likely* self-interaction inherent in the specific "external component" *relative to the studied system* can be calculated. It is the theoretical achievement that allows the analyst to try simulations of the system's evolution as it can be expected on the basis of the available data.

An indefinite number of different evolution processes may originate from every *standing state* of the studied system, depending on the features of the *initial phases* observed or hypothesized. Whereas, once the initial phase has been identified, the respective possible evolution cycle remains univocally determined.

The conclusion of each cycle, instead, depends on the analyst's choice: It may entail either a new *standing state*, achieved by the system after a transformation of its configuration and structure, or the system's disbandment because of *impossible* solutions to the equations of the evolution cycle. The analyst's choice accounts – in particular – for the phase parameters that characterize the last actual phase of the cycle in which the equation solutions are *positive numbers*.

"Mutations" in the system may also occur, when the system's *size* changes either because of new emerging components that join the exiting ones or because of components that disappear from the system for a reason whatever. In particular, the emergence of new components or the disappearance of existing ones may occur when the system approaches the maximum syntropy, or the maximum entropy, respectively, relevant to the system's size. Mutations do always entail modifications of the system's size, *i.e.,* changes in the number of the system's differentiated components and, therefore, also in the system's transformation potential.

6.1.2 *Experiencing applications of the method*

So far, there have been not many opportunities for applying the method to real situations, mainly because most applications of this method need to be based on direct surveys, which, especially in the field of urban and regional planning (that is my own field of professional practice) take relatively long time and very high implementation costs.

I can however mention a few important applications that have been useful both for suggesting response to real problems and for improving the method.

An early application of the method took place in Libya to analyse the possible impact on the overall development of Tripoli metropolitan area to be expected in consequence of the implementation of a large project of urban renewal of the city centre. The method had to use – and to adjust to – secondary data collected in preceding years by other experts for the preparation of a transportation plan for the city.[43]

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[43] *Traffic Flow System and Development Hypotheses for the Urban Area of Tripoli*, Sviluppo Tecnica SpA, Rome, 1984

The first noticeable test undergone by the method had to regard the plan for a convenient expansion process concerning the urban area of Malang, a city of East Java (Indonesia).[44] On that occasion, the application of the method was preceded by a survey on the daily all-purpose trips made by the city population and vehicles, as well as by people and vehicles from/towards surrounding regional areas in different times of an "average day" of activity. The method was used to process the survey findings and other secondary data with a view to *probing* by simulations the possible effects to be expected in consequence of the implementation of various planning choices hypothesized.

A major experience was made regarding the preparation of a strategic plan for the development of Surabaya metropolitan region (Indonesia), which entailed surveys on the regional transportation system and the use of the regional *input-output economic matrix*. This application of the method was carried out during a period of grave economic and political crisis that hit Indonesia and other South-East Asian countries[45].

Another major and high challenging opportunity for an application of the method was the evaluation of the possible impact caused by the implementation of the Metropolitan Medan Urban Development Project. It was a laborious work that had to avail itself of several special surveys on the field, followed by an onerous activity of data processing, all resulting in a technical report that consisted of six heavy volumes.[46] The applied methodology was adopted by the Asian Development Bank (the project financing institute) as a standard reference for all analogous cases.

6.2 *Theory as model*

Together with the definition of system, the theory expounded in this paper introduces a definition of *syntropy* as quantity complementary to *entropy*, the latter considered as the amount of *disorder* and/or *confusion-uncertainty* that affects the system's activity under observation.

Starting from a theoretical maximum level $H = 2\ln N$ of possible disorder, which depends on the *size N* of the system and marks the boundary level at which no system can be recognized, any lower level E of disorder (entropy) implies a degree of *order,* which is just the condition that allows an observer to recognize the existence of a system whatever.

The "balance *S.*" of "missing disorder" is assumed to be the *degree of order* or *organization* dubbed "syntropy" of the system.

Syntropy S represents the *variable* amount of disorder that the system's activity subtracts from *chaos*; so that – by definition – sum $E+S = H$ is a constant quantity *relative to the system considered*.

Then it is also clear that the order-organization may be increased by the system's activity through a continued increase in the *balance of missing disorder* subtracted from the relevant

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[44] *Simulation of Malang Physical Growth*, in co-operation with computer programmer Silvano Pellecchia (within the preparation of the Malang Urban Development Programme), C. Lotti & Ass., on behalf of BAPPEDA Malang, 1994-1995

[45] - *Towards a Policy for the Regional Development,* and

 - *Suggestions for Anti-crisis Initiatives (The Socio-economic Promotion Role of Urban Primary Activities and Services)*, BCEOM Consultants at the UMU

  BAPPEDA, , Surabaya 1996-1998

[46] In particular, concerning the performance philosophy, the volume *Model Proposed by the Logical Framework Approach*, Metropolitan Medan Urban Development Project, GKW Consult on behalf of the Ministry of Settlements and Regional Infrastructure, Jakarta, 2003-2004

extreme amount *H*. This quantity, which represents for the system the *potential disorder* at which the system disappears from observation, represents also the *transformation potential* that establishes the theoretical organization extreme that the system can achieve.

The system's "activators" are supposed to be motivated to interact by the main – though not *exclusive* – purpose of increasing the system's organization, since the theory shows that the "rewarding" for the system's activators is – on an average – coincident with the system's syntropy *S*.

The evolution process described by the proposed simulation model is the development of activities generated by a system of expectations (the *intents*), which tend to be conservative, but are necessarily modified, because of the feedback effects associated with the interactions' mutual influence, *i.e.,* by the development itself of the system's overall activity, which is expressed just by the interaction configuration.

The evolution develops by *transformation cycles*, each starting from a (supposed) condition of *precarious* equilibrium (*standing* state). Once a cycle is concluded by a transformation of both structure and configuration, a new unstable equilibrium is achieved by the system: This enters a novel *standing state* from which, unpredictably, a new transformation cycle can start. In the same way, a sequence of further cycles can follow.

If, at a certain point in the evolution, the transformation of the system's structure does not occur to adapt the set of intents-expectations to the *emerging state* of the system, then – at the conclusion of the *failed transformation* cycle – the system exits from the area of conventional reality. It should mean that the system's structure cannot bear the load of the changes that appear necessary to keep the system alive.

Looking at the definitions of entropy and syntropy and to the simulation exercises, it is easy to observe that higher levels of syntropy implies higher levels of *inequality* - between the system's components - in terms of activity concentration. In other words, the higher the differentiation between the level of activity of the system's components the higher is the degree (*S*) of organization that "rewards" the system. This merely *logical* fact might induce *political* remarks concerning the reliability of the theory, if this purports to be applicable also to socio-economic systems. Nevertheless, should socio-economic systems be addressed by application of the method suggested here, the theory would appear to be closer to real situations than different political visions are.

Hierarchy levels in *organized* systems of activities are as *natural* as *profitable*. In this connection, consider, as a major example, the formation of systems of interdependent specialized activities through which cities, civilizations and – especially – contemporary societies could emerge and develop. Obviously, there are aspects of hierarchical organization in human societies that are neither *natural* nor *spontaneous*, because imposed by force. It is the case, for example, of societies ruled by dictatorial regimes. (In the least primitive of such regimes, profitable development shared by a large part of the relevant population is still possible, though the opposite seems to be rather a frequent case)[47]. However, in most theories and models formulated to study complex systems, the *spontaneous* formation and evolution of the systems is a basic assumption, to the extent to which most of the activators (agents) of such systems are supposed *not to be subjected* to a *central control*.

Thus, should my theory **inappropriately** be considered as a possible model of the macro-functioning of human societies, an implication of the foregoing observations is that any self-organized human society is *necessarily* affected by inequalities, which differentiate groups of

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[47] Economic liberty is not necessarily associated with democracy, which in some cases suffers from heavy bureaucracy and corruption or "dogmatic" norms of a religious origin. Authoritarian regimes, in which the political power is not democratically shared, do not necessarily entail lack of economic freedom or absence of *spontaneous* incentives to free private entrepreneurial initiative.

members from one another not only according to the respective social roles and functions, but – *inevitably* – also according to the re-distribution of the system's *syntropy equivalent*, *i.e.*, in the distribution of the benefits coming from the system's overall activity.

Albeit unpleasant as it can be, and irrespective of the theory I have expounded in this paper, it is the general condition in which human societies form and develop: *Inequality* in itself seems even to be the indispensable engine of development.

Accounting for historical experience, it seems highly improbable that *equality* could co-exist with development. Inspired political visions might suggest that such a co-existence is desirable and possible. Particularly concerning the equitable distribution of the resources produced by the activities of any highly organized human society, I doubt there are persuasive theoretical or practical proofs that the co-existence is possible and beneficial.

Theories of macro-economy are rather reluctant to deal with *equitability*, preferring instead to address theorems about market-production-economic *equilibriums*, though "equilibriums" seem even more utopian than equitability, since they are apparently denied by the observed socio-economic development processes. Indeed, I deem that the most *realistic* aspect, if any such aspect may be recognized in my theory as "model", is just the proved impossibility of *permanent* equilibriums in complex self-organized and evolving systems. The concept of *stability* formulated by the theory refers just to the *probability* for a system *to remain in its present state*.

An image of acceptable and possible socio-economic development could be provided by a special reshaping and reading of the graphical curve drafted by economist Lorenz to represent the resource distribution within a social system. "Lorenz curve" is shown below by **Figure 6.1**:

Figure 6.1
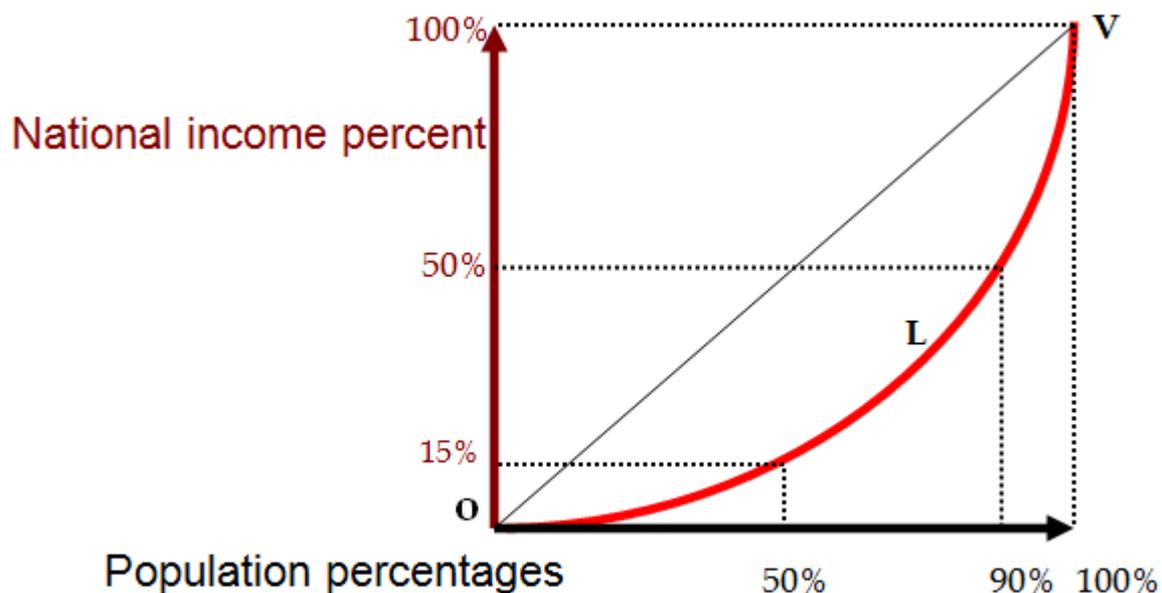
This diagram represents the concentration of resources produced in a hypothetical country, according to the geometrical curve L suggested by economist M. O. Lorenz. In the case represented, 50% of the country's population enjoys only 15% of the overall national income; 90%

of the population enjoys 50% of the national total income, while the remaining 50% income is enjoyed by only 10% of the total population.

The straight line segment OV represents the ideal situation in which the resource distribution is perfectly equitable, the income distribution being quite uniform among the country's population. As known, no such a situation has ever been observed. Thus, the distance of line L from straight line OV is an indication of the concentration of the resources: the greater the distance the higher the concentration.

According to an interpretation of the theory expounded in the preceding chapters, a higher concentration of the benefits coming from the system's overall activity is associated both with a higher efficiency of the system's organization and with a higher level of the average benefit coming from the system's activity.

Lorenz curve might not be sufficient to represent the situation concerning the actual benefit distribution among the population of any country, in that the curve shows a relation between two percentage scales, hiding what happens – for example – if the population percentage is related to the distribution of the *absolute* value of the total national income.

In this connection, suppose now that, instead of *percent* values, the axis of ordinates of the diagram in **Figure 6.1** measures the *absolute* value of the national income in two different situations; the first relevant to year 2020, when the national income amounts to one thousand billion euros, the second relative to year 2024, in which the national income is raised to 2,155 billion euros (**in real terms** with respect to the purchasing power of 2020), as represented by the following graph:

Figure 6.2
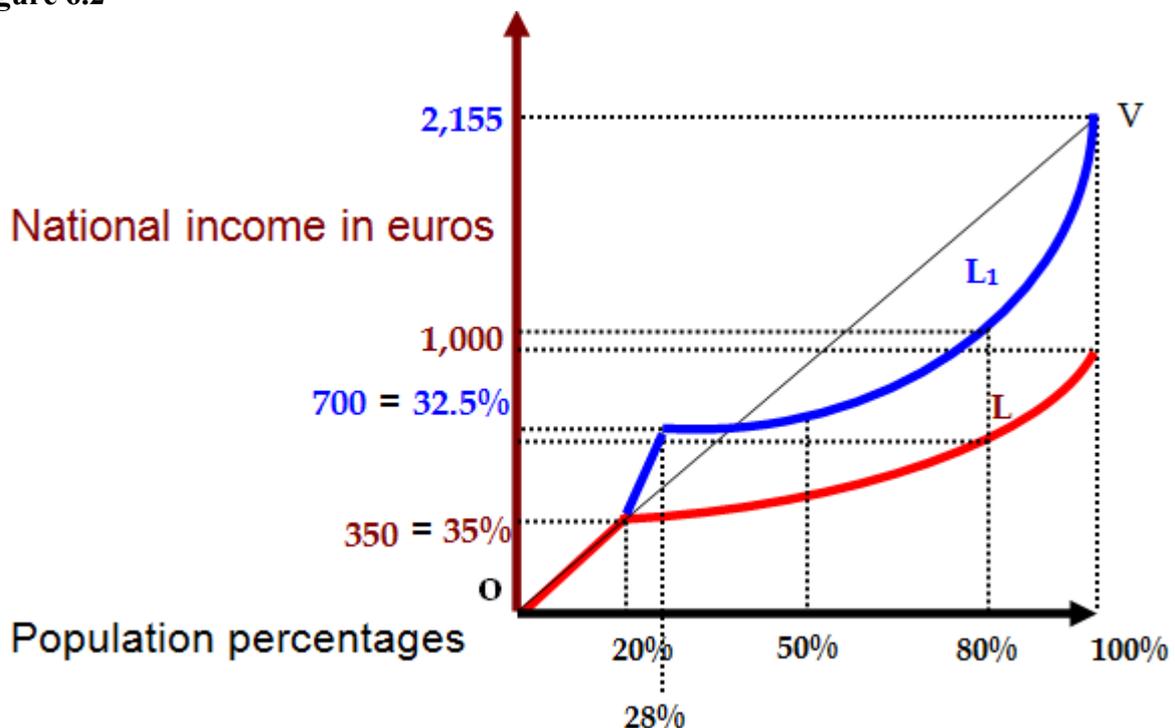
This diagram shows how it could be possible to observe an economic development in which the benefit concentration increases with the total resources produced, while raising also (from 20% to 28%) the population percentage that can share – **at constant prices** – *twice* the amount of the lower benefits enjoyed four years before (700 billion euros instead of 350; *i.e.,* on an average, 25 billion

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euros per each 1% population instead of the previous 17.5 billion euros); though the absolute value of the lower benefit section constitutes now a lower percentage of the total national income with respect to the previous percentage (32.5% instead of 35%).

The graphical scheme of **Figure 6.2** represents the possibility of a noticeable improvement (+42.8% on average) to the status of the poorer population, along with a marked increase in the *inequality* that characterizes the social system represented. In year 2024, in particular, the 20% reacher population enjoyes a larger portion (~43%) of the overall resources with respect to that (~35%) enjoyed by the 20% reacher population of four years before.

This kind of graphical scheme may be drafted in a variety of shapes, according to different possible cases, and may in general be considered as expressive of real socio-economic development processes observed.

6.3 *The trouble with knowledge*

In approching the closing of this final chapter, it seems worth making a digression, which is in my view motivated by the worldwide diffusion of an embarassing way of speaking and thinking of science.

It would be instructive to pay attention to the history of the scientific development occurred during the last eighty-ninety years.

An emerging tendency to privilege abstract mathematical theories with respect to experimentation has in particular characterized areas of physics and cosmology, after the amazing favour encountered by Einstein's General Relativity and by quantum physics.

As to cosmology, branches of this discipline have since long entered a sophisticated mathematical delirium, which impels a number of theories to stubbornly survive against observational evidence.

As to physics, under an unprecedented pressure exerted by numerous clubs of academic theorists dealing with sub-atomic worlds, quite abstract and arbitrary theories have given rise to hyper-costly experimentation projects aimed at *bending* experience to abstract theoretical expectations. All this, in the opinion of a few scientists, has had a bad influence on crowds of *brilliant* theoretical researchers that work in quite different fields: One of which is naturally the world of complex systems, from biology to economics, without excluding politics.[48]

The sin is not so much in the degree of abstraction of the theories regarded, but in enforcing the belief that just that is the correct path to *scientific* knowledge; *i.e.,* the belief that the events of the real world outside there *can* or *must* by all means be bent to the *dictate* of the theories, especially when these are not built upon any sound experimental ground.

As the entire world has recently experienced, the most dangerous mythology, by which most entrepreneurs and politicians have been and still are ill charmed, gravitates about theories of economics, sociology and finance.

Concerning economics, Ormerod wrote:

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[48] Just a few references**:** Concerning the state of physics see L. Oldershaw, *The new physics – Physical or mathematical science?*, American Journal of Physics, 56,1075, 1988; D. Lingley, *The End of Physics,* Basic Books, New York 1993; L. Smolin, *The Trouble with Physics*, Houghton Mifflin, New York 2006; P. Voit, *Not Even Wrong,* Basic Books, New York 2006.

Concerning texts that testimony to the mess that, in particular, characterizes macro-economics, see for instance: M. Friedman, *Have monetary policies failed?*, American Review, LXII, 1972; P. Ormerod, *The Death of Economics,* St. Martin's Press, London-New York 1994; J. Stiglitz, *Freefall: America, Free Markets and The Sinking of World Economy,* W. W. Norton & Co., New York 2010.

"The orthodoxy of the economics is such that this subject is taught at the universities as a *series of discovered truths* [49] concerning how the economic world *is really working*. As a matter of fact, no science could boast such a claim. […] It is not the use of mathematics in itself that leads orthodox economists along the erroneous path, but rather their ambition to compete with the authority of physics, by jointly considering the world as a well running machine. Such an erroneous view has demanded the service of mathematical instruments and methods".

Later, in 1998, fascinated by a non-deterministic model formulated by economist Alan Kirman concerning the consumer behavior with respect to a particular good,[50] the same Ormerod changed his mind (as any reader may understand) with a paper in which he purports to have found the appropriate explanation for (almost) every aspect of the mankind's behavior:
"The ideas I advance in this paper demand the use of far modern mathematics than in the case of conventional economics, which remains fixated with the maths of nineteeth-century engineers".[51]

Thus, it seems that in Ormerod's opinion the *weak point* of conventional economics does not so much reside in an imitation of physics, *i.e.,* in the pretence of explaining the world by use of mathematical instruments, but rather in *the kind* of mathematics economists use. Ormerod considers conventional economics as an old-fashioned instrument, like many might consider "mechanistic" physics. Perhaps, in Ormerod's view the new economics he proposes and supports should somehow be viewed as the analogous of *indeterministic physics* or of the relevant philosophical approach to the general socio-economic reality. Is then Ormerod's *general* representation of the world "more scientific" than the preceding ones just for he shows how to process basic assumptions by means of a "far modern" mathematics?
Besides, the reader interested in any profitable study of complex systems might be impressed by Ormerod's verbosity, typical of the 20th century's philosophers, rather than by Ormerod's inclination for new mathematical instruments. The simple basic scheme of the model he adopts is certainly an interesting and legitimate methodological option, especially if his theory, beyond mathematial formulas, propose viable examples or ways of objectively useful applications. Until such applications are repeatedly and satifactorily tested on the field, his "general theory of the world society and economy" remains only one of the more or less beautiful *explanations for everything*, offered to the public attention in an *idle* competition with a host of other metaphysical theories and religions.

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[49] The Italic font is in the original text, **The Death of Economics,** *op. cit.*, pp. 28 and 125.
[50] Kirman's model, which concerns fish market, is recalled in a recent book of this author, **Complex Economics. Individual and Collective,** Routledge, New York 2011
[51] P. Ormerod, **Butterfly Economics. A General Theory of Social and Economic Behavior,** Faber & Faber Ltd., London 1998, Introduction, p. X

Closing

Despite all possible criticisms, the profound pulsion to find justifiable explanations for the myriad of complex events that condition human lifes, as well as the unceasing quest for effective instruments to control such events, remain unrenounceable human aspirations, which not only must be respected and admired but also encouraged to manifest in any reasonable way.

However, we should never lose the awareness that pre-selected theoretical options, cultural formation and prejudices do always bias the way in which we tend to represent the reality we observe, even when our basic purpose is to avoid any reference to particular philosophical criteria or principles. Representation is necessarily simplification, as positive science has taught us usefully. The problem is *what* and *how* can usefully be simplified.

Yet, I do believe that the fate of our knowledge would even worsen if theories and models concerning the behaviour of human societies, instead of striving for simplification, could identify, assess and incorporate *all* the imaginable variables and parameters that we consider as important**:** An insoluble problem would arise, because of the impossibility to establish in principle the *unquestionable* way in which we shall put so many variables and parameters in relation with each other.

An attempt of that sort was made by the Club of Rome in years Seventy to predict the evolution and fate of the mankind, but – as known – the attempt failed. The connections between the very many variables were established by deterministic criteria following Forrester's school[52] of thought and by the adoption of substantially arbitrary parameters, which proved inadequate.[53] Attempts of a very similar kind, partially connected with the activities of the Club of Rome, regard another group of *complex systems*, which are addressed by the models of ecosystems aimed at predicting the destiny of our planet's climate and much more. The enjoyable result of such efforts consists in a remarkable confusion (strongly and obviously denied by the model builders), according to which almost *anything* – and the relevant opposite – may be *predicted*.

From another side of our contemporary world, a host of super-skilled magnificent experts in computational languages have persuaded many serious scholars that computers and computer games can profitably replace laboratory and on-the-field experimentation, especially to the benefit of those disciplines that are actually excluded from viable and replicable experimental procedures.

On the one hand, undoubtely, most games are important sources of knowledge since the first days of each individual life. Contemporary sophisticated games and simulations carried out by the aid of computers are often an effective means to accustom humans with major unusual aspects of

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[52] Jay W. Forrester (1919) is considered as the "father" of *Systems Dynamics*; is the author of many papers and books on systems dynamics, starting from 1961 to date. His book **Urban Dynamics**, Pegasus Communications, MIT Cambridge 1969, is of a particular interest. "*System dynamics is a professional field that deals with the complexity of systems. System dynamics is the necessary foundation underlying effective thinking about systems. System dynamics deals with how things change through time, which covers most of what most people find important. System dynamics involves interpreting real life systems into computer simulation models that allow one to see how the structure and decision-making policies in a system create its behaviour",* from **System Dynamics: the Foundation under Systems Thinking,** MIT, Cambridge, June 1999.

[53] Club of Rome, **The Limits to Growth,** 1972 ISC Annual Management Symposium in S. Gallen. Prepared mainly by Donella and Dennis Meadows, it is a model of the world development that consists of about thousand equations formulated according to the principles and criteria proper to the "system dynamics" thaught by Jay Forrester. The model was followed in 1973-1974 by another one of the same nature, prepared by E. Pestel and M. Mesarovic, adopted by the Club of Rome with the title **The Mankind at a Turning Point**, which allows for about two hundred thousand (*sic!*) equations.

important technological and scientific activities, though such *games* and simulations can work well as training activities, certainly not as scientific instruments for exploring the "unknown reality".

On the other hand, if it is to any extent significant and useful availing oneself of computerized simulations that obey rigorous, tested or testable theories of positive science or logic, there is instead to seriously question the service rendered to the progress of effective knowledge by what is only playing computerized *games*, which both *imitate* the real world *at will*, and *still at will* produce pictures of imaginary worlds by means of quite special computer performances.

I do not know how many of us are aware of the cultural, social and political risks we incur by replacing Middle-Age myths and rites with the computational ones of the present time.